# Group-Strategyproof Irresolute Social Choice Functions

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Theorem (Gibbard, 1973; Satterthwaite, 1975): A strategyproof resolute SCF is either imposed or dictatorial.

> *[Resoluteness] is a rather restrictive and unnatural assumption.*
>
> Peter Gärdenfors (philosopher)

> *The Gibbard-Satterthwaite theorem [...] uses an assumption of singlevaluedness which is unreasonable.*
>
> Jerry S. Kelly (economist)

> *If there is a weakness to the Gibbard-Satterthwaite theorem, it is the assumption that winners are unique.*
>
> Alan D. Taylor (mathematician)

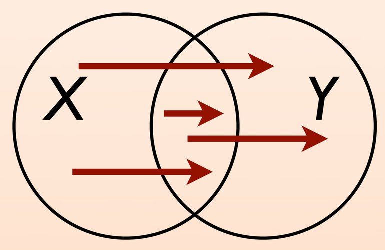
## Preliminaries

- Each voter $i$ has a complete preference relation $R_i$ over a finite set of at least three alternatives.
- A **social choice function (SCF)** is a function that maps a preference profile to a non-empty subset of alternatives.
- An SCF $f$ is **resolute** if $|f(R)|=1$ for all preference profiles $R$.
- An SCF is (weakly) **strategyproof** if no voter can obtain a more preferred outcome by misrepresenting his (strict) preferences.
- An SCF is **group-strategyproof** if no group of voters can obtain an outcome that all of them prefer to the original one.
- An SCF is **pairwise** if it only depends on the difference of the number of voters who prefer $x$ to $y$ and those who prefer $y$ to $x$ for every pair of alternatives $x$ and $y$.
  - Examples: Kemeny, Borda, Maximin, ranked pairs, all tournament solutions (Slater set, uncovered set, Banks set, minimal covering set, bipartisan set, TEQ, etc.)
- An SCF satisfies **set-monotonicity** if weakening unchosen alternatives has no effect on the choice set.

## Kelly's Preference Extension

Underlying assumption:
Nothing known about tie-breaking mechanism

$X\ R_i\ Y \Leftrightarrow \forall x \in X, y \in Y: (x\ R_i\ y)$




Example: $a\ R_i\ b\ R_i\ c \Rightarrow \{a\}\ R_i\ \{a,b\}\ R_i\ \{b,c\}$

$\{a,c\}$ and $\{b\}$ are incomparable

## Related Work

- Theorem (Barbera, 1977; Kelly, 1977): Every strategyproof quasi-transitively rationalizable SCF is either imposed or dictatorial.
- However, quasi-transitive rationalizability itself is highly problematic.
  - e.g., Gibbard (1969), Schwartz (1972), Mas-Colell et al. (1972)
  - "one plausible interpretation of such a theorem is that, rather than demonstrating the impossibility of reasonable strategy-proof social choice functions, it is part of a critique of the regularity [rationalizability] conditions" (Kelly; 1977)
  - "whether a nonrationalizable collective choice rule exists which is not manipulable and always leads to nonempty choices for nonempty finite issues is an open question" (Barbera; 1977)
- Various negative results for stronger set extensions (e.g., Duggan and Schwartz; 2000)

## Results

- Theorem 1: No Condorcet extension is strategyproof.
- Theorem 2: Every SCF that satisfies set-monotonicity is weakly group-strategyproof.
- Theorem 3: Every weakly strategyproof, pairwise SCF satisfies set-monotonicity.
- Corollary: A pairwise SCF is weakly group-strategyproof iff it satisfies set-monotonicity.

## Consequences & Discussion

- Our main result can be seen as an irresolute version of the Muller-Satterthwaite theorem.
- *SP* (strategyproofness): resistance vs. preference misrepresentation
  - *PA* (participation): resistance vs. abstention
  - *SSP* (strong superset property): resistance vs. adding/deleting losing alternatives
  - *CC* (composition consistency): resistance vs. cloning alternatives

| | *Strategic manipulation* | | *Agenda manipulation* | |
|---|---|---|---|---|
| | *SP* | *PA* | *SSP* | *CC* |
| Plurality | - | ✓ | - | - |
| Borda | - | ✓ | - | - |
| Copeland | - | - | - | - |
| MC | ✓ | ✓ | ✓ | ✓ |
| BP | ✓ | ✓ | ✓ | ✓ |

- MC and BP can be computed efficiently.

| *group-strategyproof* | *manipulable* |
|---|---|
| Pareto rule | essentially everything else |
| Omninomination rule | |
| Top cycle | |
| **Minimal covering set (MC), 1988** | |
| **Bipartisan set (BP), 1993** | |
| **Tournament equilibrium set (TEQ), 1990** [subject to 20-year old conjecture] | |

## Proof of Theorem 1

R

| 2 | 2 | 2 | 1 | 1 | 1 |
|---|---|---|---|---|---|
| bc | ac | ab | b | c | a |
| | | | c | a | b |
| a | b | c | a | b | c |

wlog: $b \in f(R)$

R'

| 2 | 1 | 1 | 2 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|
| bc | a | ac | ab | b | c | a |
| | c | | | c | a | b |
| a | b | b | c | a | b | c |

Case 1: $b \notin f(R') \Rightarrow$ Red voter manipulates ($R$ ➠ $R'$)

Case 2: $b \in f(R')$

R''

| 2 | 1 | 1 | 2 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|
| bc | a | a | ab | b | c | a |
| | c | c | | c | a | b |
| a | b | b | c | a | b | c |

Condorcet: $\{a\}=f(R'') \Rightarrow b \notin f(R)$

$\Rightarrow$ Blue voter manipulates ($R'$ ➠ $R''$)

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This material is based on work supported by the Deutsche Forschungsgemeinschaft under grants BR 2312/3-3 and BR 2312/7-1.

July 2011