

Random Assignment with Optional Participation

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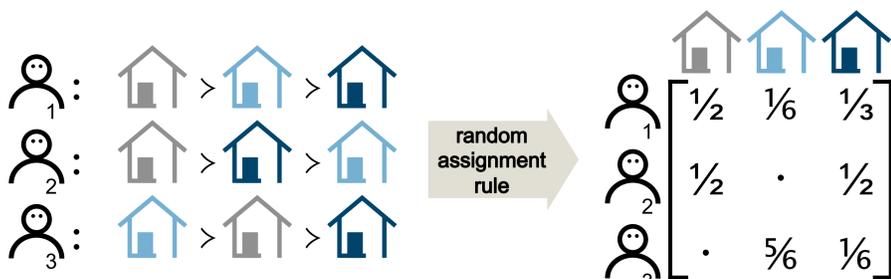
Preliminaries

Assignment problem (A, H, \succ) : n agents have preferences over n houses

Deterministic assignment or matching M : n pairwise disjoint $(agent, house)$ -tuples

Random assignment p : Probability distribution over deterministic assignments. $p_{a,h}$ is the probability agent a receives for house h

Random assignment rule: Selects one (or multiple) random assignments for every assignment problem



An assignment problem and the random assignment yielded by Random Serial Dictatorship.

Assume Agent 1 can choose to either **participate** or **abstain**.

- If he participates, he receives $\frac{1}{2}$ House 1 + $\frac{1}{6}$ House 2 + $\frac{1}{3}$ House 3.
- If he abstains, he receives House 1.

The former is strictly preferred to the latter according to **stochastic dominance (SD)**.

Modeling Participation

Two ways to define abstention:

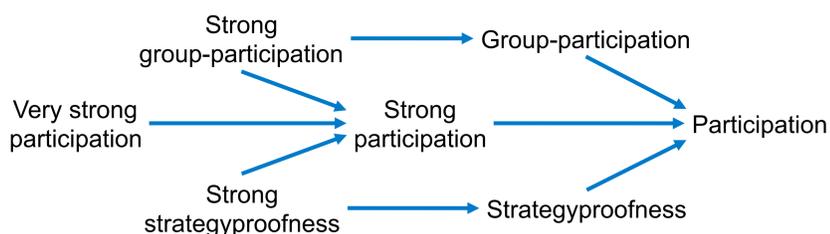
- Indifference** Absent agents are treated as if they were completely indifferent.
 - 1) Houses are assigned to participating agents according to their preferences.
- Two rounds**
 - 2) Remaining probability shares are distributed uniformly to absent agents.

Both interpretations are equivalent for the random assignment rules considered.

Defining Incentives

Three notions of SD-participation [Brandl et al., 2015]:

- Participation** Participating cannot yield a worse assignment.
- Strong participation** Participating always yields a (weakly) better assignment.
- Very strong participation** Participating always yields a strictly better assignment (whenever possible).



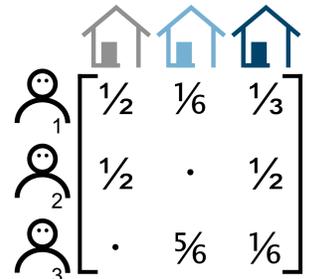
Logical relations between different notions of participation and strategyproofness.

Four Important Random Assignment Rules

Random Serial Dictatorship (RSD):

- Select a sequence of agents uniformly at random.
- In this order, every agent picks his most preferred remaining house.

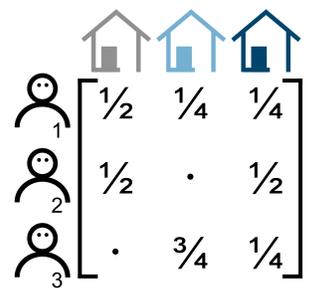
- RSD satisfies very strong participation.
- RSD satisfies strong group-participation.



(Extended) Probabilistic Serial (PS):

- Each agent reserves probability of his first indifference class of houses at uniform speed.
- When a set of houses is completely reserved, agents continue with their most preferred still available indifference class.

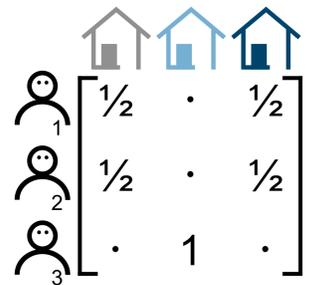
- PS satisfies very strong participation.
- PS satisfies strong group-participation.



(Naive) Boston Mechanism (BM):

- In the i^{th} round, every remaining house is assigned to an agent ranking it at i^{th} place uniformly at random.
- All assigned agents and houses leave the procedure after each round.

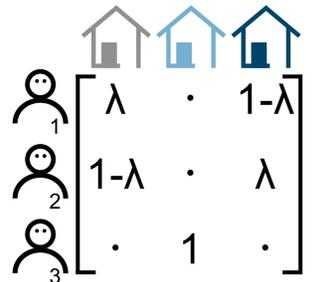
- BM satisfies very strong participation.
- BM satisfies group-participation.



Popular random assignment rules (PRA):

- Random assignments such that there does not exist another random assignment which is majority-preferred.
- Correspond to (weak) Condorcet-winners in the majority graph.

- PRA satisfy group-participation.



	Very strong participation	Strong group-participation	Group-participation
Random Serial Dictatorship	✓	✓	✓
Probabilistic Serial	✓	✓	✓
Boston Mechanism	✓	—	✓
Popular random assignment rules	—	—	✓

Summary and Conclusion

Results are largely positive, participation (basically) always incentivized, in many cases even strictly. Even if identifying or communicating one's preferences involves minor costs, agents are better off participating than abstaining.

Open problem: Do PRA satisfy strong participation?

References: F. Brandl, F. Brandt, and J. Hofbauer. Incentives for participation and abstention in probabilistic social choice. In *Proc. of the 14th AAMAS*, 1411–1419. IFAAMAS, 2015.