Arrovian Aggregation via Pairwise Utilitarianism

Extended Abstract

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We consider Arrovian aggregation of preferences over lotteries that are represented by skew-symmetric bilinear (SSB) utility functions, a significant generalization of von Neumann-Morgenstern utility functions due to Fishburn, in which utility is assigned to pairs of alternatives. We show that the largest domain of preferences that simultaneously allows for independence of irrelevant alternatives and Pareto optimality when comparing lotteries based on accumulated SSB welfare is a domain in which preferences over lotteries are completely determined by ordinal preferences over pure alternatives. In particular, a lottery is preferred to another lottery if and only if the former is more likely to return a preferred alternative. Preferences over pure alternatives are unrestricted. We argue that SSB welfare maximization for this domain constitutes an appealing probabilistic social choice function.

A central concept in welfare economics are Arrow social welfare functions (SWFs), i.e., functions that map a collection of individual preference relations over some set of candidates to a social preference relation over the candidates. Arrow's seminal impossibility theorem states that every SWF that satisfies Pareto optimality and independence of irrelevant alternatives is dictatorial (Arrow, 1951). It has been shown that this theorem still holds when the set of candidates consists of all lotteries over some finite set of alternatives and individual preferences over lotteries satisfy the von Neumann-Morgenstern axioms, i.e., preferences over lotteries can be represented by assigning cardinal utilities to alternatives and comparing lotteries based on expected utility (Arrow et al., 2002, Chapter 10, Theorem 4.3).\textsuperscript{1}

\textsuperscript{1}See also Sen (1970); Kalai and Schmeidler (1977); Hylland (1980) for variants of this theorem.
In this paper, we consider Arrovian aggregation of preferences over lotteries under loosened assumptions about preferences over lotteries. In particular, we assume that preferences over lotteries are given by \textit{skew-symmetric bilinear (SSB) utility functions}, which assign utilities to every \textit{pair} of lotteries. These functions are assumed to be skew-symmetric and linear in both parameters. SSB utility theory is a generalization of linear expected utility theory due to von Neumann and Morgenstern (1947), which does not require the controversial independence axiom and transitivity (see, e.g., Fishburn, 1982, 1984b, 1988). The independence axiom requires that if lottery $p$ is preferred to lottery $q$, then a coin toss between $p$ and a third lottery $r$ is preferred to a coin toss between $q$ and $r$ (with the same coin used in both cases). There is experimental evidence that both of these axioms are violated systematically in real-world decisions. The Allais Paradox (Allais, 1953) is perhaps the most famous example pointing out violations of independence. A detailed review of such violations is provided by Machina (1983). Mas-Colell et al. (1995, p. 181) conclude that “because of the phenomena illustrated […] the search for a useful theory of choice under uncertainty that does not rely on the independence axiom has been an active area of research.” Even the widely accepted transitivity axiom seems too demanding in some situations. For example, the preference reversal phenomenon\(^2\) (see, e.g., Grether and Plott, 1979) shows failures of transitivity. SSB utility theory assumes neither independence nor transitivity and can accommodate both effects, the Allais Paradox and the preference reversal phenomenon. Fishburn (1982) gives an axiomatic characterization of SSB preferences using three intuitive axioms: continuity, dominance, and symmetry. The latter two are weakenings of independence and transitivity. For a more thorough discussion of SSB utility theory, the reader is referred to Fishburn (1988).

Despite a lack of transitivity, the Minimax Theorem (von Neumann, 1928) implies that, for every SSB utility function and every compact and convex set of lotteries, there is a maximal lottery, i.e., a lottery that is weakly preferred to any other lottery within the set. In other words, the main appeal of transitivity—the existence of maximal elements—remains intact. We show that choosing maximal elements from every compact and convex set satisfies standard choice consistency conditions, namely Sen’s $\alpha$ (contraction) and $\gamma$ (expansion) (Sen, 1971). SSB preference relations can thus be interpreted as rationalizing relations for the choice behavior of rational agents.

The key question we pursue in this paper is which domains of SSB preferences over lotteries allow for Arrovian aggregation, i.e., for SWFs that satisfy both Pareto optimality and independence of irrelevant alternatives. A set of lotteries is considered feasible if and only if it is the convex hull of some subset of alternatives. We restrict attention to SWFs that compare lotteries with respect to accumulated SSB welfare and, because of the pairwise nature of SSB utilities, refer to this as \textit{pairwise utilitarianism}. Our main result is that the largest neutral domain that allows for Arrovian aggregation is one where preferences over lotteries are completely determined by ordinal preferences over alternatives

\(^2\)The preference reversal phenomenon prescribes that lottery $p$ is preferred to lottery $q$, but the certainty equivalent of $p$ is lower then the certainty equivalent of $q$. 

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in the following sense: an agent prefers lottery \( p \) to lottery \( q \) if and only if it is more likely that the alternative returned by \( p \) is preferred to the alternative returned by \( q \) than the other way round.

These preferences over lotteries are quite natural and can be seen as the canonical SSB representation consistent with a given ordinal preference relation over alternatives (see Blavatskyy (2006) for an axiomatic characterization and Aziz et al. (2015, 2016) for a study of efficiency and strategyproofness with respect to such preference relations). It is particularly noteworthy that the characterized domain allows for arbitrary preference relations over alternatives and thus encapsulates well-known domains that allow for Arrovian aggregation in the deterministic context (such as dichotomous and single-peaked preferences). The probabilistic social choice function that returns maximal lotteries for a given profile of individual preferences over alternatives was proposed by Fishburn (1984a) and recently axiomatized using population-consistency and composition-consistency (Brandl et al., 2016).

An interesting question is how much the assumption of welfare maximization can be weakened. For the case when agents’ preferences over lotteries admit a linear expected utility representation, welfare maximization is readily justified: Harsanyi (1955) shows that even the seemingly innocent assumption that society is indifferent between two lotteries whenever all individuals are indifferent between them leaves weighted welfare maximization as the only possibility. Notice that this is a statement about a single preference profile considered in isolation. The weights given to the agents may depend on their preferences. This can be prevented by adding axioms that connect the collective preferences across different profiles. The SWF that derives the collective preferences by adding up the normalized utility representations is known as relative utilitarianism. It was characterized by Dhillon and Mertens (1999) using essentially independence of redundant alternatives (a weakening of independence of irrelevant alternatives) and monotonicity (a weakening of a Pareto-type axiom). As shown by Fishburn and Gehrlein (1987) and further explored by Turunen-Red and Weymark (1999), aggregating SSB utility functions is fundamentally different from aggregating vNM utility functions in that Harsanyi’s Pareto-type axiom does not imply weighted welfare maximization.

**References**


