On the Incompatibility of Efficiency and Strategyproofness in Randomized Social Choice

Haris Aziz
NICTA and UNSW

Florian Brandl
TU München

Felix Brandt
TU München

Randomized Social Choice

A **social decision scheme (SDS)** maps a preference profile to a **lottery** (probability distribution) over alternatives. Two desirable properties of SDSs are

- **Efficiency:** no agent can be made better off without making another agent worse off, and
- **Strategyproofness:** no agent can benefit from misrepresenting his preferences.

Related Work

Gibbard (1973) & Satterthwaite (1975): Every single-valued, Pareto-optimal, and strategyproof social choice function is a dictatorship.

Gibbard & Sonnenschein (1977): Every Pareto optimal and strongly *SD*-strategyproof SDS is a probability mixture of dictatorships.

Bogomolnaia & Moulin (2001): There is no anonymous, neutral, *SD*-efficient, and strongly *SD*-strategyproof SDS.

Preliminaries

Each **agent** i has a complete and transitive **preference relation** R_i over a set of **alternatives** A. A **social decision scheme (SDS)** maps a preference profile $(R_1,\ldots,R_n)$ to a **lottery** p over A. An SDS is

- **anonymous** if it is symmetric with respect to agents.
- **neutral** if it is symmetric with respect to alternatives.
- **pairwise** if the outcome depends on the pairwise majority comparisons only.

Lottery Extensions

Many reasonable lottery extensions can be defined.

Bilinear dominance (*BD*):
$p\ R_i^{BD}\ q$ if for all x, y, where $x\ P_i\ y$, $\quad p(x)q(y) \geq p(y)q(x)$.

Stochastic dominance (*SD*):
$p\ R_i^{SD}\ q$ if for all x, $\quad \sum_{y R_i x} p(y) \geq \sum_{y R_i x} q(y)$.

Pairwise comparison (*PC*):
$p\ R_i^{PC}\ q$ if for all x, y, $\quad \sum_{x P_i y} p(x)q(y) \geq \sum_{x P_i y} q(x)p(y)$.

Downward/Upward lexicographic (*DL*/*UL*):
$p\ R_i^{DL}\ q$ if there is x such that $p(x) > q(x)$ and for all $y\ P_i\ x$, $p(y) = q(y)$.
$p\ R_i^{UL}\ q$ if there is x such that $p(x) < q(x)$ and for all $x\ P_i\ y$, $p(y) = q(y)$.

Example: $a\ P_i\ b\ P_i\ c$;

$p = \frac{2}{3}a + \frac{1}{3}c$; $q = b$; and $r = \frac{2}{3}b + \frac{1}{3}c$.

Then $p\ P_i^{PC}\ q$; $p\ P_i^{DL}\ q$; $p\ P_i^{SD}\ r$; $q\ P_i^{BD}\ r$; and $q\ P_i^{UL}\ p$.

$$R_i^{BD} \subseteq R_i^{SD} \subseteq R_i^{PC}, \qquad R_i^{SD} \subseteq R_i^{DL}, \qquad R_i^{SD} \subseteq R_i^{UL}$$

Efficiency and Strategyproofness

Let $E \in \{BD, SD, PC, DL, UL\}$ be a lottery extension.

- In a preference profile R, a lottery p is ***E*-efficient** if there is no q such that $q\ R_i^E\ p$ for all i and $q\ P_i^E\ p$ for some i. An SDS f is E-efficient if $f(R)$ is E-efficient for every preference profile R.
- An SDS f is **Pareto-optimal** if $f(R)$ does only support Pareto-optimal alternatives for every R.
- An SDS f is ***E*-strategyproof** if there are no R, R', i such that $R_j = R'_j$ for all $j \neq i$ and $f(R')\ P_i^E\ f(R)$.
- An SDS f is **strongly *E*-strategyproof** if $f(R)\ R_i^E\ f(R')$, whenever $R_j = R'_j$ for all $j \neq i$.

Impossibility Theorems

- There is no anonymous, neutral, *PC*-efficient, and *PC*-strategyproof SDS.
- There is no anonymous, *UL*-efficient, and *UL*-strategyproof SDS.*
- There is no pairwise, Pareto-optimal, and *BD*-strategyproof SDS.**
- There is no anonymous, neutral, Pareto-optimal, and *BD*-group-strategyproof SDS.***

Conjecture: There is no anonymous, *SD*-efficient, and *SD*-strategyproof SDS.

Anonymity is required for all our results as **serial dictatorship** satisfies all considered notions of efficiency and strategyproofness.

*Interestingly, random dictatorship satisfies anonymity, neutrality, *DL*-efficiency, and *DL*-strategyproofness.

**This strengthens a result by Aziz, Brandt, and Brill (2013).

***This strengthens a result by Bogomolnaia, Moulin, and Stong (2005).

Technische Universität München

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