Group-Strategyproof Irresolute Social Choice Functions

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Theorem (Gibbard, 1973; Satterthwaite, 1975): A strategyproof resolute SCF is either imposed or dictatorial.

Preliminaries

- Each voter \( i \) has a complete preference relation \( R_i \) over a finite set of at least three alternatives.
- A social choice function (SCF) is a function that maps a preference profile to a non-empty subset of alternatives.
- An SCF \( f \) is resolute if \( |f(R)| = 1 \) for all preference profiles \( R \).
- An SCF is (weakly) strategyproof if no voter can obtain a more preferred outcome by misrepresenting his (strict) preferences.
- An SCF is group-strategyproof if no group of voters can obtain an outcome that all of them prefer to the original one.
- An SCF is pairwise if it only depends on the difference of the number of voters who prefer \( x \) to \( y \) and those who prefer \( y \) to \( x \) for every pair of alternatives \( x \) and \( y \).
  - Examples: Kemeny, Borda, Maximin, ranked pairs, all tournament solutions (Slater set, uncovered set, Banks set, minimal covering set, bipartisan set, TEQ, etc.)
- An SCF satisfies set-monotonicity if weakening unchosen alternatives has no effect on the choice set.

Related Work

- Theorem (Barbera, 1977; Kelly, 1977): Every strategyproof quasi-transitively rationalizable SCF is either imposed or dictatorial.
- However, quasi-transitive rationalizability itself is highly problematic.
  - e.g., Gibbard (1969), Schwartz (1972), Mas-Colell et al. (1972)
  - “one plausible interpretation of such a theorem is that, rather than demonstrating the impossibility of reasonable strategy-proof social choice functions, it is part of a critique of the regularity [rationalizability] conditions” (Kelly, 1977)
  - “whether a non-rationalizable collective choice rule exists which is not manipulable and always leads to non-empty choices for non-empty finite issues is an open question” (Barbera, 1977)
- Various negative results for stronger set extensions (e.g., Duggan and Schwartz, 2000)

Consequences & Discussion

- Our main result can be seen as an irresolute version of the Muller-Satterthwaite theorem.
- \( SP \) (strategyproofness): resistance vs. preference misrepresentation
  - \( PA \) (participation): resistance vs. abstention
  - \( SSP \) (strong superset property): resistance vs. adding/deleting losing alternatives
  - \( CC \) (composition consistency): resistance vs. cloning alternatives

Kelly’s Preference Extension

Underlying assumption: Nothing known about tie-breaking mechanism

\[ X, R, Y \Rightarrow \forall x \in X, y \in Y: (x, y) \]

Example: \( \{a, b, c\} \)

Related Work

Proof of Theorem 1

- Theorem 1: No Condorcet extension is strategy-proof.
- Theorem 2: Every SCF that satisfies set-monotonicity is weakly group-strategyproof.
- Theorem 3: Every weakly strategyproof, pairwise SCF satisfies set-monotonicity.
- Corollary: A pairwise SCF is weakly group-strategyproof iff it satisfies set-monotonicity.