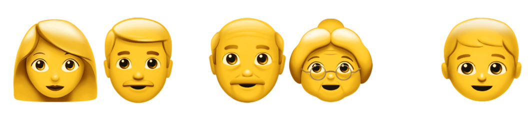
Technische Universität München

An Analytical and Experimental Comparison of Maximal Lottery Schemes

Florian Brandl, Felix Brandt, Christian Stricker

Maximal Lottery Schemes

Step 1: Construct the matrix of pairwise majority margins.



a	b	c
b	c	a
c	a	b

	a	b	c
a	0	1	-1
b	-1	0	3
c	1	-3	0

Step 2: Apply an odd and monotone function τ to the majority margins.

$C1\text{-}ML\colon \tau = \mathrm{sgn}$

$C2\text{-}ML\colon \tau = \mathrm{id}$

	a	b	c
a	0	τ(1)	τ(-1)
b	τ(-1)	0	τ(3)
c	τ(1)	τ(-3)	0

Step 3: Compute maximin strategies (mixed Nash equilibria) of the resulting two-player zero-sum game.

$$C1\text{-}ML(R) = \left\{\frac{1}{3}a + \frac{1}{3}b + \frac{1}{3}c\right\} \qquad C2\text{-}ML(R) = \left\{\frac{3}{5}a + \frac{1}{5}b + \frac{1}{5}c\right\}$$

Efficiency & Strategyproofness

A **lottery extension** extends preferences over alternatives to (possibly incomplete) preferences over lotteries of alternatives.

- Stochastic Dominance (*SD*):
  $p \succsim^{SD} q$ **if and only if** $\sum_{y \succsim x} p(y) \geq \sum_{y \succsim x} q(y)$ **for all** $x \in A$
- Pairwise Comparison (*PC*):
  $p \succsim^{PC} q$ **if and only if** $\sum_{x \succsim y} p(x)q(y) \geq \sum_{y \succsim x} p(x)q(y)$

Efficiency: No other lottery is weakly *SD*-preferred (*PC*-preferred) by all agents and strictly by one agent.

Strategyproofness: No agent can get an *SD*-preferred (*PC*-preferred) lottery by misrepresenting his preferences.

Positive Results

Every C2-ML scheme is ***PC*-efficient**.

Every ML scheme is ***PC*-strategyproof** in every profile that admits a Condorcet winner.

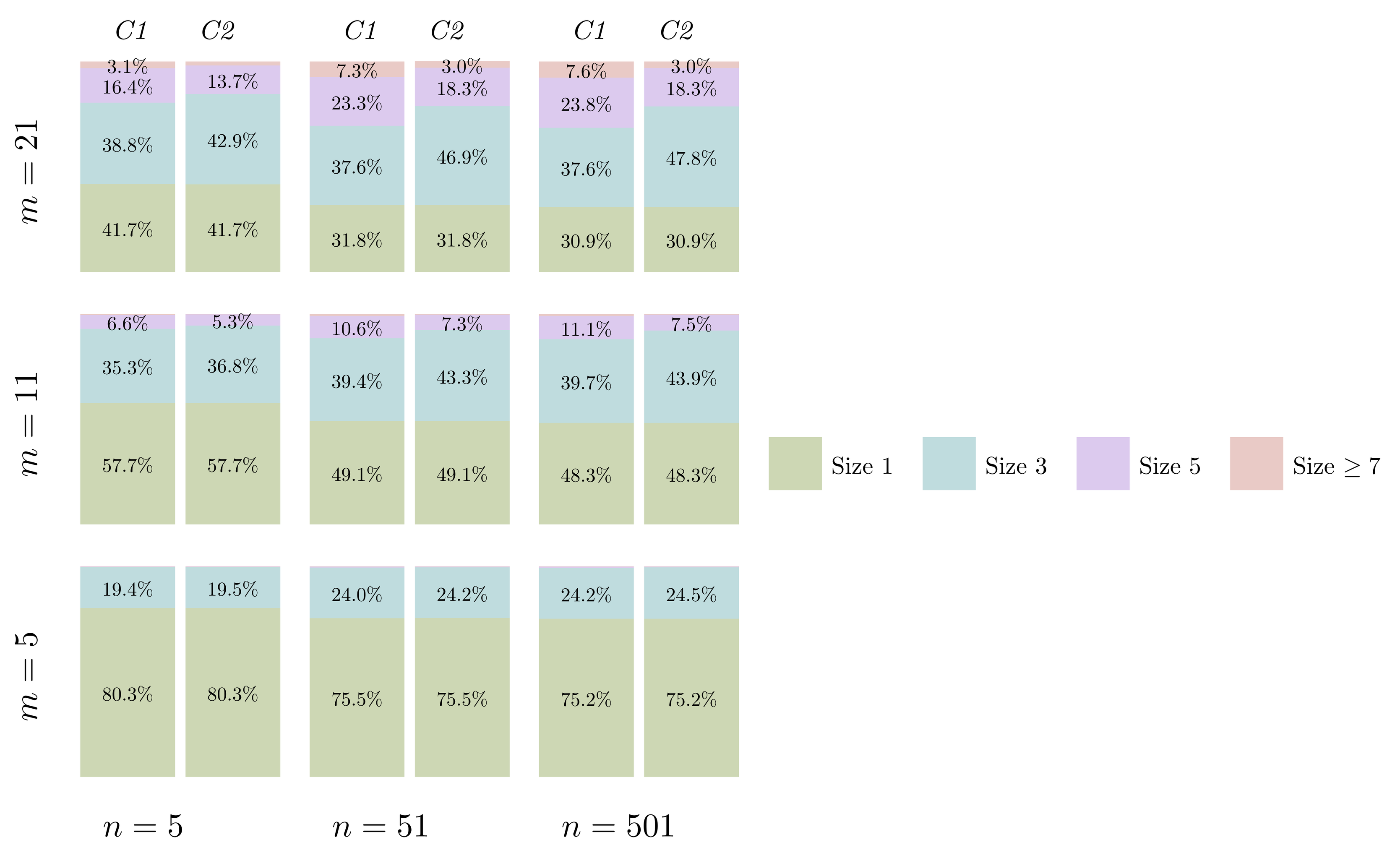
Negative Results

No ML scheme except C2-ML is ***SD*-efficient**.

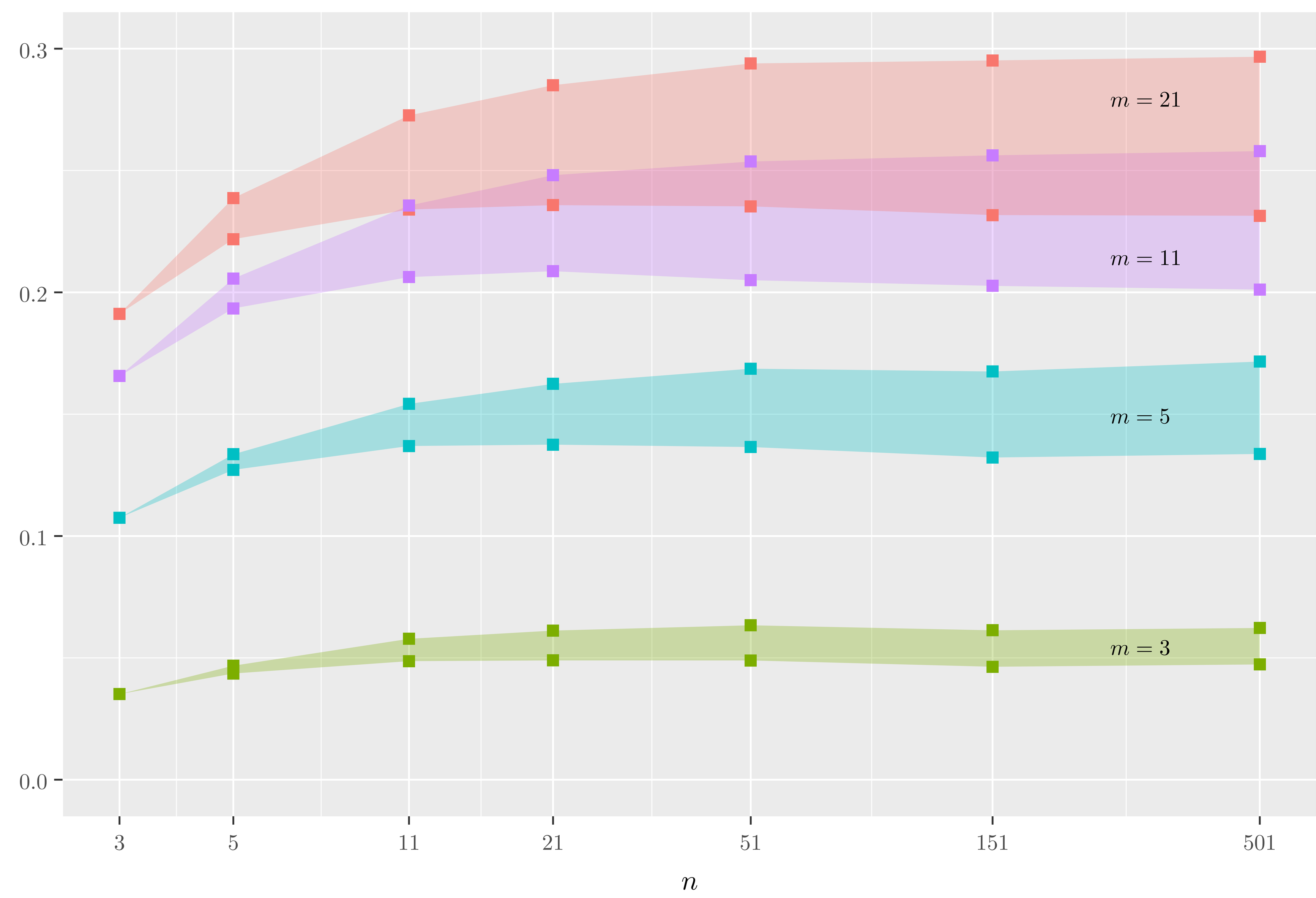
No ML scheme is ***SD*-strategyproof**.

Every C2-ML scheme is ***SD*-manipulable** in every fully diverse profile that does *not* admit a weak Condorcet winner.

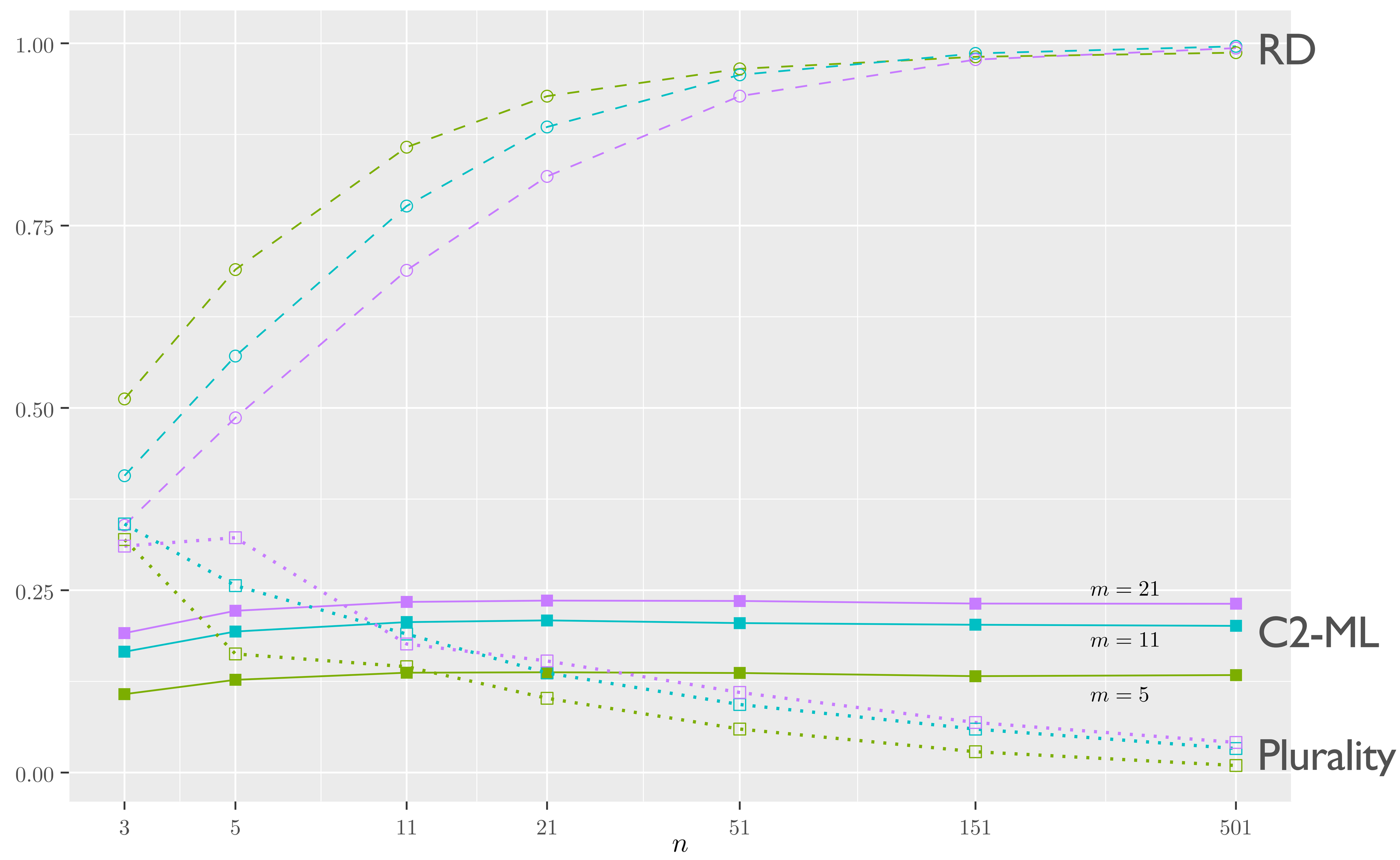
Degree of Randomization



Average support size under IAC (C1-ML vs. C2-ML)



Shannon entropy under IAC (C1-ML vs. C2-ML)



Shannon entropy under IAC (C2-ML vs. Random Dictatorship and Plurality)


votation.ovh


voting.ml


pnyx.dss.in.tum.de