

Exploring auction-based leveled-commitment contracting

Part III: Vickrey-type auctioning

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Abstract

A key problem addressed in the area of multiagent systems is the automated assignment of multiple tasks to executing agents. The automation of multiagent task assignment requires that the individual agents (*i*) use a common protocol that prescribes how they have to interact in order to come to an agreement and (*ii*) fix their final agreement in a contract that specifies the commitments resulting from the assignment on which they agreed. The work reported in this paper is part of a broader research effort aiming at the design and analysis of approaches to automated multiagent task assignment that combine auction protocols and leveled commitment contracts. The primary advantage of such approaches is that they are applicable in a broad range of realistic scenarios in which knowledge-intensive negotiation among agents is not feasible and in which unforeseeable future environmental changes may require agents to breach their contracts. Examples of standard auction protocols are the English auction, the Dutch auction, and the Vickrey auction. In [2, 3] combinations of English/Dutch-type auctioning and leveled commitment contracting have been described. In this report the focus is on the combination of Vickrey-type auctioning and leveled commitment contracting.

1 Introduction

The area of multiagent systems (e.g., [8, 12, 23]), which is concerned with systems composed of technical entities called agents that interact and in some sense can be said to be intelligent and autonomous, has achieved steadily growing interest in the past decade for two major reasons. First, it provides innovative methods and concepts for designing, realizing, and handling modern—distributed, large-scale, dynamic, open, and heterogeneous—information processing systems. The Internet is just the most prominent example of such systems; others are multi-database systems and in-house information systems. Second, it offers useful technology for developing and analyzing models and

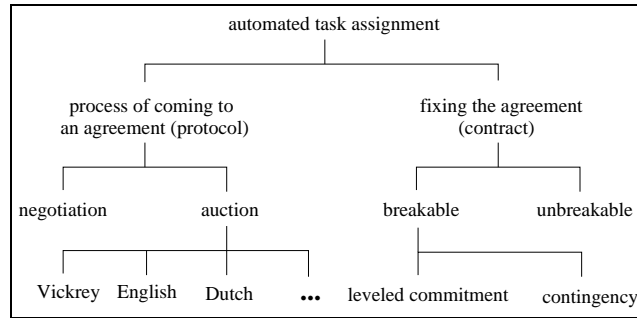


Figure 1: Automated task assignment.

theories of interactivity among humans. Humans, like other intelligent natural beings, do not function in isolation, but interact in various ways and at various levels; however, the relationships between intelligence and interactivity are still poorly understood.

A key problem addressed in this area is the automated assignment of multiple tasks to executing agents under criteria such as efficiency and reliability. The automation of task assignment requires that the agents *(i)* use a common protocol that prescribes how they have to interact in order to come to an agreement on “who does what” and *(ii)* are willing to fix their final agreement in a formal or “legally valid” contract. The protocol concerns the act or process of finding an appropriate task assignment, while the contract concerns the consequences and commitments resulting from the assignment on which the agents agreed. Two standard types of task assignment protocols are negotiation-based protocols (e.g., [5, 11, 20]) and auction-based protocols (e.g., [4]). Examples of widely applied auction protocols are the English auction, the Dutch auction, and the Vickrey auction (e.g., [14]). Compared to negotiation-based protocols, auction-based protocols show several distinct and advantageous features: they are easily implementable, they enforce an efficient (low-cost and/or low-time) assignment process, and they guarantee an agreement even in scenarios in which the agents possess only very little domain- or task-specific knowledge. Two standard types of task assignment contracts are unbreakable contracts (e.g., [10, 15, 16]) and breakable contracts, where common forms of breakable contracts are contingency contracts (e.g., [13]) and leveled commitment contracts (e.g., [1, 6, 18, 19]). Compared to unbreakable contracts, breakable contracts offer a significant advantage: they allow agents acting in dynamic environments to flexibly react upon future environmental changes that make existing contracts unfavorable. Figure 1 summarizes this rough overview of available approaches to automated task assignment.

This report describes work that aims at investigating automated task assignment in multiagent systems that combines auction-based protocols and breakable contracts. More specifically, an approach to multiagent task assignment is introduced that is based on a Vickrey-type auction protocol *and* leveled commitment contracting. The advantage of such a combination is that it is applicable in a very broad range of realistic scenarios in which knowledge-intensive

negotiation among agents is not feasible and in which future environmental changes may require agents to breach their contracts. In [2, 3] combinations of English/Dutch-type auctioning and leveled commitment contracting have been described. Basic descriptions of Vickrey auctions, also known as second-price sealed bid auctions, can be found in e.g. [14, 21]; a more general discussion of the advantages and limitations of this auction protocol is provided in [17]. Compared to other auction protocols, Vickrey auctions have the advantage that their duration is a priori known (each interested agent bids only once) and that the dominant bidding strategy is to bid one’s true valuation. This makes Vickrey auctions particularly interesting for applications in computational settings (see e.g. [7, 9, 22]).

This work is structured as follows. Section 2 describes the contracting framework, and Section 3 explains our variation of the Vickrey auction. Section 4 presents experimental results that indicate the benefits of this approach. Finally, Section 5 concludes the report with a brief overview of potential research directions evoked by the idea of combining auctioning and leveled commitment contracting.

2 Auction-Based Contracting (ABC)

We consider a group of agents that contains two different types of business partners. *Contractors* CR_i ($i = 1 \dots m$) who offer a unique task i and *Contractees* CE_j ($j = 1 \dots n$) who are willing to execute tasks. A contractor CR_i is capable of executing task i by himself for his prime costs¹ $C[CR_i]$. A contractee CE_j is able to do each task i for $C[CE_j, i]$.

We assume that contractees can accomplish tasks cheaper than contractors by defining two intervals.

$$\begin{aligned} \forall i : C[CR_i] &\in [cr_{min}, cr_{max}] \\ \forall j, i : C[CE_j, i] &\in [ce_{min}, ce_{max}] \\ ce_{max} &\leq cr_{min} \end{aligned}$$

This ensures that both, contractors and contractees, are interested in signing contracts with each other.

Pursuing conflicting goals, both types of agents are “true capitalists”: contractors intend to pay the lowest feasible price for a task, while contractees try to earn as much money as possible.

During an auction round, each contractor offers his task, where the contractor sequence randomly varies from round to round. Applying an auction-based protocol the agents then come to an agreement which contractee will execute the task. A contractee is only able to accept *one* task per round. For this reason, we consider two basic types of contract obligation: *full commitment* (a contractee has to stay with the first deal he made) and *leveled commitment* (contractors can breach contracts by paying a fine $Penalty_j$ to the concerning contractor CR_i).

¹All prices and bids are integers.

We investigate two types of penalty. The first is defined as a fraction of the contract value $P[i]$, the second is a fraction of the contractor’s task costs.

$$\textit{Price penalty: } \textit{Penalty}_j = \textit{ppr} \cdot P[i]$$

$$\textit{Cost penalty: } \textit{Penalty}_j = \textit{cpr} \cdot C[CR_i]$$

\textit{ppr} and \textit{cpr} are constants called price penalty rate and cost penalty rate, respectively. After buyer CE_j and seller CR_i negotiated a price $P[i]$ for task i , profits of both agent types are defined as follows.

$$\begin{aligned} CR_i : \textit{Profit}_i &= C[CR_i] - P[i] \\ CE_j : \textit{Profit}_j &= P[i] - C[CE_j, i] - \textit{PenaltySum}_j \end{aligned}$$

$\textit{PenaltySum}_j$ is the sum of penalties CE_j paid during one round.

3 Vickrey-type auction

The ABC system described in the previous section works with any (auction-based) protocol that defines how the agents have to interact to come to an agreement. The protocol investigated in this work is a variation of the Vickrey auction.

Whenever a contractor announces a task, each interested contractee calculates one sealed bid and informs the announcing contractor. The contractee who submitted the lowest bid is declared as the winner of the auction, and the *second lowest* bid is taken as the price of the announced task; the contractor pays this price to the winning contractee who in turn executes the task. (If there are two or more equal winning bids, the winner is picked randomly.) This kind of auctioning can be viewed as an “inverse variant” of the standard *Vickrey auction* in which the contractee submitting the highest bid for goods or resources wins the auction at the second highest bid. (This is why the kind of auctioning described here is called *Vickrey-type* auction.) Vickrey-type auctioning is appealing for computational applications for two main reasons. First, the winner is determined after just one bidding cycle; obviously this is particularly useful in time- and/or cost-sensitive domains. Second, the dominant strategy in Vickrey auctions is to bid one’s true value; obviously this is desirable because it helps to avoid wasteful counterspeculation in a broad range of competitive buyer-seller settings.

In order to take into consideration that usually contractees are limited in their capacity, it is assumed that each contractee can not be involved in more than one contract at the same time. (This assumption could be easily relaxed such that a contractee can not be simultaneously involved in $c \in N$ contracts.) As an extension of “pure auctioning,” however, each contractee is allowed to decommit from a contract by simply paying a *decommitment penalty* to the corresponding contractor. This enables a contractee to legally breach a contract whenever there is a more profitable task announcement. The penalty specification are part of the contracts. In particular, the penalties are assumed to

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FOR Round = 1 TO Max_Rounds
  CR_Set := {CR_i: i = 1 ... m};
  CE_Set := {CE_j: j = 1 ... n};
  WHILE CR_Set ≠ {} AND CE_Set ≠ {}
    CR := Choose_random(CR_Set);
    CR_Set := CR_Set \ CR;
    i := 1;
    FOREACH c ∈ CE_Set
      bid[i] := c.Bid(CR.Task);
      c[i] := c;
      i := i+1;
    ENDFOREACH
    CE := c[Index_of_Minimum(bid[])];
    Sort_ascending(bid[]);
    Contract(CR,CE,bid[2]);
    IF Commitment = full THEN CE_Set = CE_Set \ CE;
  ENDWHILE
ENDFOR

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Figure 2: Conception of the Vickrey-type auction and contracting framework

be variable and not conditioned on future events; this kind of breakable contracts are known as *leveled commitment contracts*, in contrast to contingency contracts. The level of commitment is determined by the amount of penalty to be payed for breaching. With that, the task assignment approach described in this paper combines standard-type auctioning with a highly flexible form of contracting.

Figure 2 shows the basic algorithm of the auction. The set of contractees must be passed through in random order because there might be two or more contractees that accept the same offer. In this case, the winner has to be picked randomly.

3.1 Bidding Details

There is a whole spectrum of possible bidding strategies. The realization described in the following has been chosen because it is intuitively clear, easily extensible, and efficiently realizable. Whenever a contractor CR_i initiates a new auction by announcing his task, each potential contractee CE_j calculates his bid. This calculation is done as follows. If CE_j is not already involved in another contract in the current auction round, then his bid is given by

$$Bid_j = (1 + dp_{ji}) \cdot C[CE_j, i] \quad (1)$$

where dp_{ji} is a variable factor called desired profit (of contractee CE_j w.r.t. the tasks announced by contractor CR_i). Whenever a contractee CE_j wins an auction for a task announced by a contractor CR_i , he raises the factor dp_{ji}

according to

$$dp_{ji} = (1 + IncreaseInit_j) \cdot dp_{ji} \quad (2)$$

where $IncreaseInit_j$ is a contractee-specific constant. This ensures that a contractee who wins an auction initiated by some contractor i will submit a higher bid in the next auction initiated by this contractor and thus tries to further increase his future profit. Whenever CE_j does not win an auction initiated by CR_i , then he reduces dp_{ji} according to

$$dp_{ji} = (1 - DecreaseInit_j) \cdot dp_{ji} \quad (3)$$

where $DecreaseInit_j$ is a contractee-specific constant. The situation is somewhat more sophisticated if CE_j is already involved in a contract signed with another contractor CR_k . In this case CE_j additionally takes into consideration the difference $P[k] - C[CE_j, k]$ (i.e., his potential gain from the already existing contract) and the penalty $Penalty_j$ (i.e., the penalty he would have to pay for decommitting from this contract). Formally, under the assumption that CE_j is already committed to CR_k in the current auction round, CE_j calculates his bid for a task announced by CR_i as follows:

$$Bid_j = \max\{(1 + dp_{ji}) \cdot C[CE_j, i], C[CE_j, i] + P[k] - C[CE_j, k] + Penalty_j\} \quad (4)$$

where dp_{ji} is defined as above. (Note that according to the above definitions a contractee decommits from a contract only if the new contract would result in a higher profit.)

4 Experimental Results

The purpose of the experiments described here was to achieve a basic understanding of effects of combining Vickrey-type auctioning and leveled commitment contracting. All results presented in this section are based on the following parameter setting (for all i and j): $dp_{ji} = 0.1$ (i.e., initially each contractee intend to make 10% profit), $IncreaseInit_j = 0.1$, and $DecreaseInit_j = 0.1$. At the beginning of each round none of the potential contractees is involved in a contract and all penalties $Penalty_j$ are set to zero. Other parameters are chosen as described below. In the following several scenarios are investigated, differing in the number of contractors and contractees.

A number of further experiments with varying parameter settings and varying numbers of contractors and contractees (including a 32+40 scenario) have been performed; the results obtained qualitatively coincide with those reported in this work.

4.1 3 Contractors and 4 Contractees (“3+4 Scenario”)

Table 1 shows the prime costs of three contractors and six contractees. The table entries (i.e., the agents’ prime costs $C[CR_i]$ and $C[CE_j, i]$) are chosen from the intervals defined by the parameters $ce_{min} = 10$, $ce_{max} = 99$, $cr_{min} = 100$, and $cr_{max} = 200$. In this subsection a “3+4 scenario” is considered, consisting of the three contractors and the first four contractees shown in this table.

	Task 1	Task 2	Task 3
CR_1	196	–	–
CR_2	–	193	–
CR_3	–	–	115
CE_1	42	68	53
CE_2	22	46	46
CE_3	24	27	59
CE_4	12	11	19
CE_5	31	64	37
CE_6	65	24	55

Table 1: Cost table for the 3+4 and 3+6 scenario.

The Tables 2 and 3 summarize results obtained for the 3+4 scenario w.r.t. the profits accumulated by the contractors and the contractees in 100 rounds for different commitment levels (i.e., full commitment and different price/cost penalty rates).

Three interesting observations follow from these results. A first key observation with these data is that leveled commitment contracting is much fairer than full commitment contracting in that contractees having lower prime costs can effectively make more profit, in relative terms, than contractees having higher prime costs. In particular, the data clearly show that this fairness is correlated with the level of commitment. This can be most easily seen by comparing the profits made by CE_4 who is the “best” among all contractees (he can accomplish each task for the cheapest price) with the profits of the other contractees: the profits made by CE_1 to CE_3 decreases with the level of commitment, while the profit of CE_4 changes only slightly. More precisely, as can be inferred from Table 2, the ratio between CE_4 ’s profit and the sum of the other contractees’ profits is 0.82 for full commitment, while this ratio is equal to 0.86 (0.99, 1.87) for $ppr = 1.00$ ($ppr = 0.50$, $ppr = 0.25$) and equal to 0.91 (0.93, 1.79) for $cpr = 0.15$ ($cpr = 0.10$, $cpr = 0.05$). This is also illustrated by the Figures 3, 4 and 5. A second key observation is that competition among both the contractees and the contractors significantly increases as the level of commitment decreases. This can be immediately seen by comparing the overall profit made by the contractees and the contractors for different commitment levels (see the last column in each of the Tables 2 and 3). In particular, this observation indicates that the use of this task assignment scheme does have an enormous, global effect on the dynamics in electronic markets (price/cost developments) occupied by self-interested, non-cooperative agents like the contractees and contractors considered here. The Figures 6, 7 and 8, which show how the prices develop under different commitment levels, further illustrate this observation. (Prices for tasks not sold in an auction round are assumed to be zero in these figures; this ensures that only prices paid by the contractors are taken into consideration.) These figures show that leveled commitment contracting, compared to full commitment contracting, results in an obvious price pressure and

Scenario	Commitment		Broken	Accumulated Profit						$\sum_j CE_j$
				CE_1	CE_2	CE_3	CE_4	CE_5	CE_6	
3+4	full	(no penalty)	—	97	816	2,037	2,420	—	—	5,370
		price penalty	4	79	869	1,724	2,286	—	—	4,958
	leveled	ppr=1.00	42	36	603	1,467	2,081	—	—	4,187
		ppr=0.50	74	0	335	753	2,035	—	—	3,123
		ppr=0.25	0	99	650	1,896	2,398	—	—	5,043
		cpr=0.15	11	104	677	1,432	2,067	—	—	4,280
full	cost penalty	50	0	416	741	2,076	—	—	3,233	
	(no penalty)	—	0	132	16	1,776	632	224	2,780	
3+6	full	(no penalty)	0	0	145	13	1,808	573	269	2,808
		price penalty	13	0	138	3	1,601	398	182	2,322
	leveled	ppr=1.00	29	0	137	4	1,569	117	188	2,015
		ppr=0.50	0	0	150	25	1,703	638	189	2,705
		ppr=0.25	0	0	138	33	1,656	662	191	2,680
		cpr=0.15	11	0	136	0	1,597	251	207	2,191
full	cost penalty	—	0	136	0	1,597	251	207	2,191	
	(no penalty)	—	0	136	0	1,597	251	207	2,191	

Table 2: Number of broken contracts and contractees' profits accumulated in 100 rounds in the 3+4 and 3+6 scenarios for different commitment levels.

Scenario	Commitment		Accumulated Profit				
			CR_1	CR_2	CR_3	$\sum_i CR_i$	
3+4	full	(no penalty)		16,704	14,152	6,120	36,976
	leveled	price penalty	ppr=1.00	15,918	14,625	6,315	36,858
			ppr=0.50	10,848	14,258	6,554	31,660
			ppr=0.25	10,188	10,544	6,711	27,443
	leveled	cost penalty	cpr=0.15	16,691	14,420	6,227	37,338
			cpr=0.10	15,026	15,057	6,302	36,385
cpr=0.05			12,148	12,529	6,681	31,358	
3+6	full	(no penalty)		17,161	16,556	6,801	40,518
	leveled	price penalty	ppr=1.00	17,162	16,507	6,936	40,605
			ppr=0.50	15,089	16,582	7,220	38,891
			ppr=0.25	12,669	16,263	7,649	36,581
	leveled	cost penalty	cpr=0.15	17,172	16,567	6,871	40,610
			cpr=0.10	17,172	16,594	6,801	40,567
cpr=0.05			15,380	16,573	7,445	39,398	

Table 3: Contractors’ profits accumulated in 100 rounds in the 3+4 and 3+6 scenarios for different commitment levels.

thus typically in lower prices for competitive tasks, that is, for tasks that could be accomplished at low costs by several contractees. The reason behind this is that contractees already involved in other contracts contribute to the decrease of task prices whenever they participate in auctions. For instance, the prices for the tasks 2 and 3 are much lower compared to full commitment contracting because contractee CE_4 (the “best” contractee) now participates in auctions even after having signed a contract. A third key observation is that there is *no* remarkable difference between price- and cost-oriented penalty (fairness effects can be achieved with both). This indicates that the choice of the penalty mode is not crucial, as long as the penalty mode chosen allows to flexibly decommit from contracts. This observation does have an impact on the design of any assignment schemes based on level commitment contracting.

4.2 3 Contractors and 6 Contractees (“3+6 Scenario”)

In order to investigate what happens if the competition increases, two additional contractees were added to the 3+4 scenario (see Table 1). The results for this 3+6 scenario are also summarized in the Tables 2 and 3. (Figures showing the detailed price and profit curves for this scenario are not included for reasons of limited space.) These results show, in particular, that an increase in the competition results in lower prices and therefore in lower profits of the contractees and higher profits of the contractors (compared to the 3+4 scenario). All key observations mentioned above for the 3+4 scenario obviously do also hold for the 3+6 scenario. All in all, the results show that the computational approach described in the preceding section in fact realizes what is intuitively expected by “Vickrey-type leveled commitment contracting.”

5 Conclusions

Automated task assignment that combines auction-based protocols and leveled commitment contracting defines a promising field of research in the area of multiagent systems. The results show, among other things, that this combination results in a very flexible assignment scheme that shows desirable fairness properties w.r.t. the profits that can be made by the contractees. An important issue in applying this assignment scheme is that a decrease of the level of commitment results not only in an increase of the level of fairness, but also in an increase of the communication costs. This indicates that this scheme must be applied carefully in domains in which communication costs and bandwidth are critical parameters. The work described in this report and in [2, 3] is best understood as the first step toward a more comprehensive understanding of the limitations and benefits of combining auctioning and leveled commitment contracting. There are several open research issues that remain to be addressed in the future:

- Formal analysis (based on the broad range of available theoretical work on auctioning) of price stability and convergence.
- The extension of the proposed approach toward scenarios in which both the contractees and the contractors are allowed to breach contracts.
- The extension toward parallel auctions.
- The extension toward multi-unit and combinatorial auctions.
- The extension toward learning agents and more adaptive protocols.

We think that the importance of automated task assignment in multiagent systems, the broad applicability range of multiagent task assignment based on auctioning and leveled commitment contracting, and the encouraging initial experimental results and key observations reported in this paper justify to explore these and related issues.

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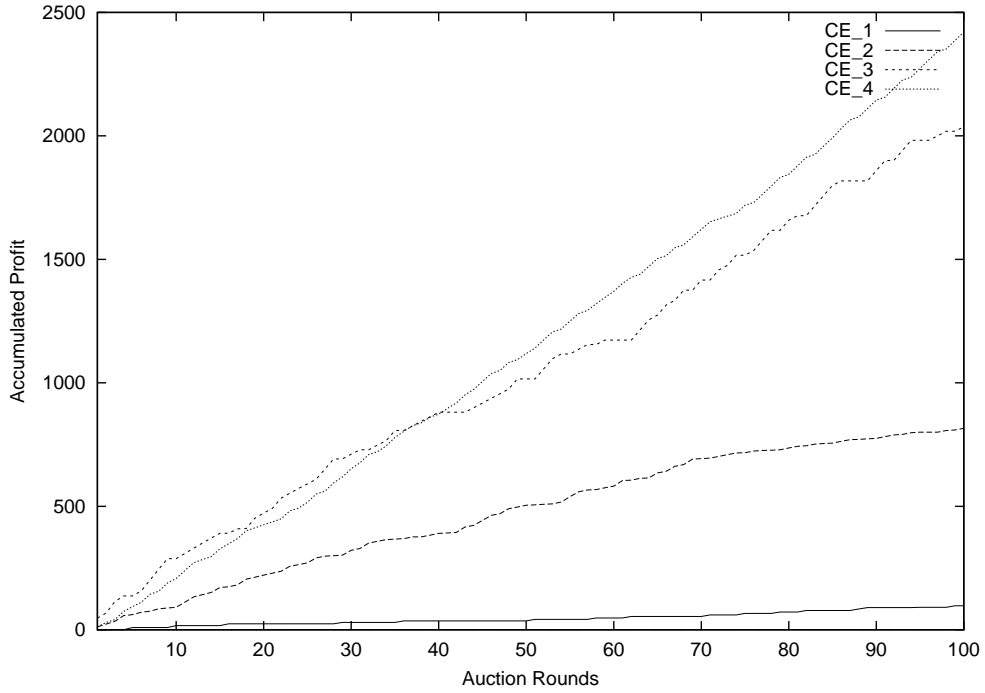


Figure 3: Accumulated profit in the 3+4 scenario with full commitment contracting.

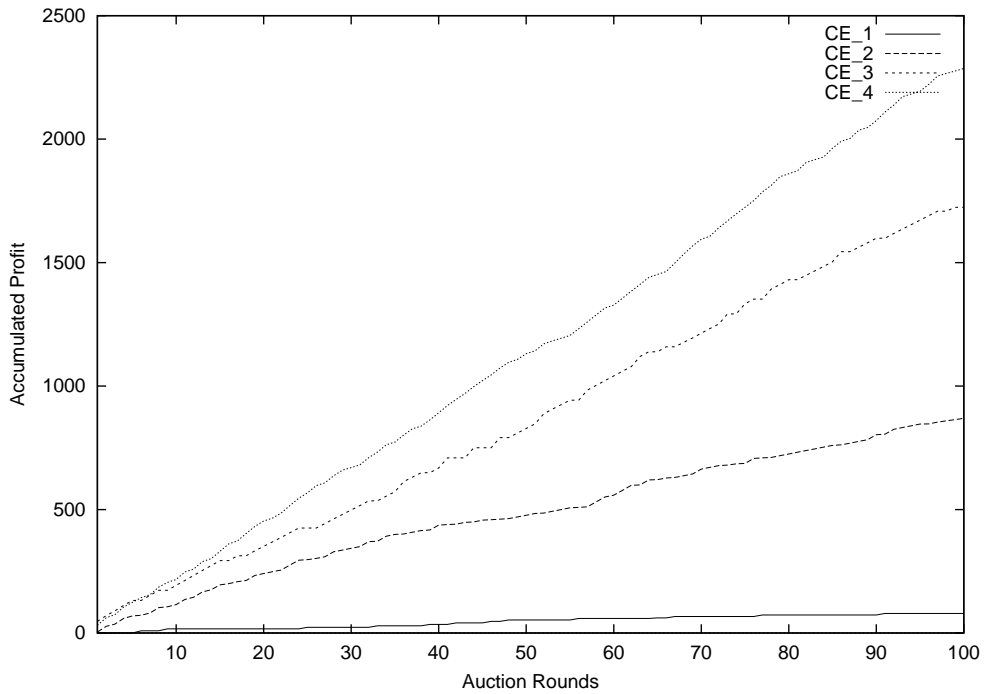


Figure 4: Accumulated profit in the 3+4 scenario with price penalty $ppr = 1.00$.

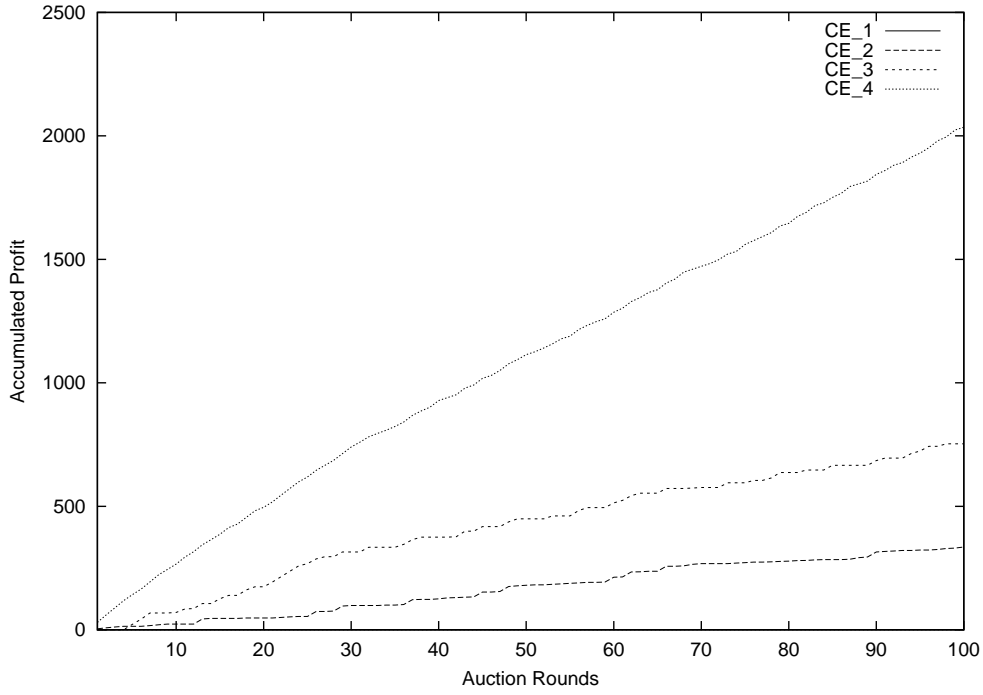


Figure 5: Accumulated profit in the 3+4 scenario with price penalty $ppr = 0.25$.

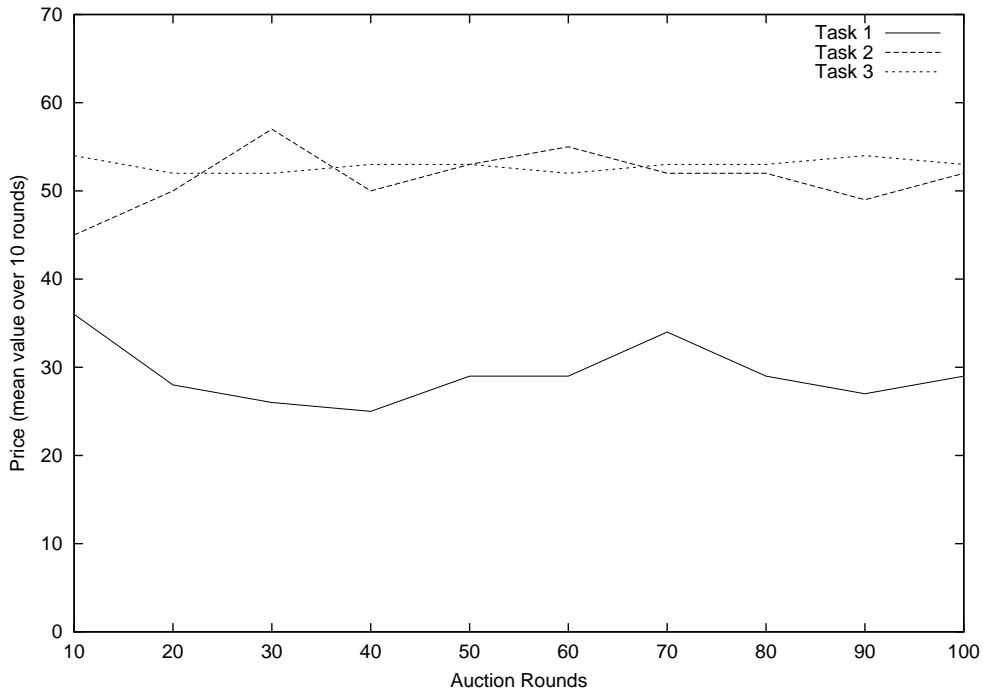


Figure 6: Price development in the 3+4 scenario with full commitment contracting.

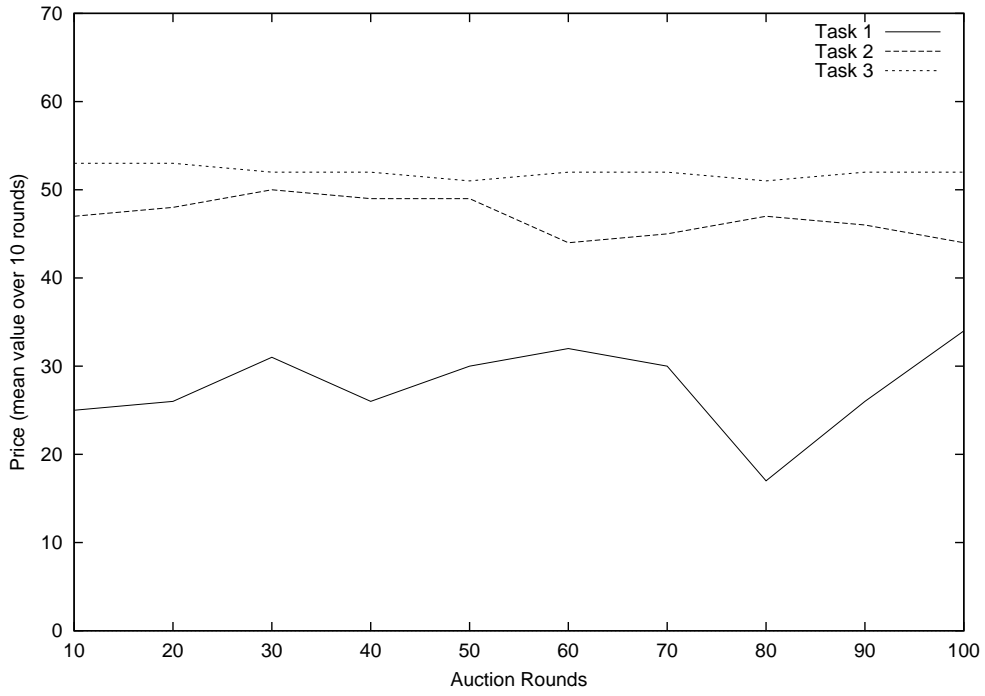


Figure 7: Price development in the 3+4 scenario with price penalty $ppr = 1.00$.

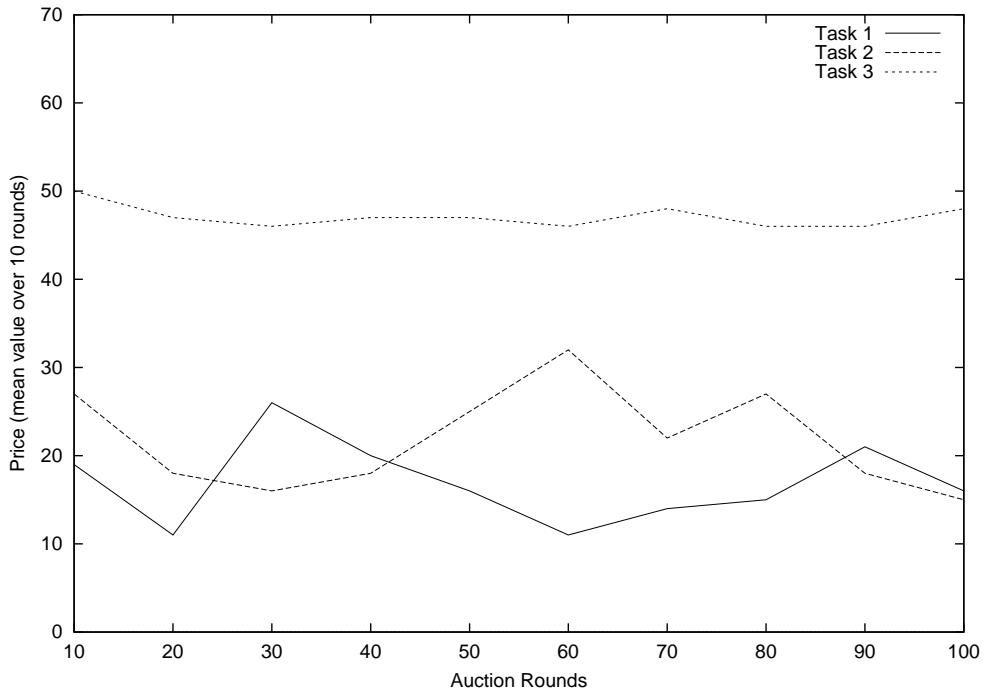


Figure 8: Price development in the 3+4 scenario with price penalty $ppr = 0.25$.