Synergistic Valuations and Efficiency in Spectrum Auctions

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Abstract

In spectrum auctions, bidders typically have synergistic values for combinations of licenses. This has been the key argument for the use of combinatorial auctions in the recent years. Considering synergistic valuations turns the allocation problem into a computationally hard optimization problem that generally cannot be approximated to a constant factor in polynomial time. Ascending auction designs such as the Simultaneous Multiple Round Auction (SMRA) and the single-stage or two-stage Combinatorial Clock Auction (CCA) can be seen as simple heuristic algorithms to solve this problem. Such heuristics do not necessarily compute the optimal solution, even if bidders are truthful. We study the average efficiency loss that can be attributed to the simplicity of the auction algorithm with different levels of synergies. Our simulations are based on realistic instances of bidder valuations we inferred from bid data from the 2014 Canadian 700MHz auction. The goal of the paper is not to reproduce the results of the Canadian auction but rather to perform "out-of-sample" counterfactuals comparing SMRA and CCA under different synergy conditions when bidders maximize payoff in each round. With "linear" synergies, a bidder's marginal value for a license grows linearly with the total number of licenses won, while with the "extreme national" synergies, this marginal value is independent of the number of licenses won unless the bidder wins *all* licenses in a national package. We find that with the extreme national synergy model, the CCA is indeed more efficient than SMRA. However, for the more realistic case of linear synergies, SMRA outperforms various versions of CCA that have been implemented in the field including the one used in the Canadian 700MHz auction. Overall, the efficiency loss of all ascending auction algorithms is small even with high synergies, which is remarkable given the simplicity of the algorithms.

Key words: Market design, Spectrum auctions, Combinatorial auctions, Simulation experiments

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1. Introduction

Radio spectrum is a key resource in the digital economy. Recognizing its enormous value for society, the US Federal Communication Commission (FCC) decided in 1994 to replace their bureaucratic process ("beauty contest") with a market-based approach to assign spectrum: the Simultaneous Multiple Round Auction (SMRA). Since then the SMRA has successfully been used by many regulators and has generated hundreds of billions of dollars worldwide. Despite this success, the SMRA has also led to a number of strategic problems for bidders. Telecom operators typically have preferences for certain packages of licenses. In the SMRA this leads to the so-called *exposure problem*: bidders who compete aggressively for a certain package risk ending up with only a subset, possibly paying more than what this subset is worth to them. The inability to express preferences for packages directly adds strategic complexity for bidders and is a source of inefficiency in the SMRA. The exposure problem that is inherent to item-by-item competition has stirred interest in combinatorial auctions, which allow bidders to submit preferences for combinations or packages directly. The design of combinatorial spectrum auctions has drawn significant attention from researchers from various fields including economics, game theory, operations research, and computer science, see e.g. Cramton et al. (2006).

Combinatorial auctions have also drawn interest from regulators. For their 2008 700MHz auction, the FCC decided to augment the SMRA with the possibility to bid on a national package. This simple combinatorial auction was based on the Hierarchical Package Bidding (HPB) format that had been designed and tested by Goeree and Holt (2010). In the same year, the British regulator Ofcom pioneered the Combinatorial Clock Auction (Cramton, 2008) and regulators world-wide have since followed their example by adopting different versions of the CCA. The single-stage CCA (Porter et al., 2003) is a simple ascending format where bidders can submit multiple package bids in each round and prices are increased on items for which there is excess demand. Variants of single-stage CCA have been used in Romania in 2012 and in Denmark in 2016. The single-stage CCA creates incentives for demand reduction (Ausubel et al., 2014), a problem which the two-stage CCA tries to address. The two-stage CCA allows for only a single package bid in each round and adds a sealed-bid "shoot out" phase and a core-selecting payment rule (Cramton, 2013). Both phases are governed by a revealed-preference activity rule. The two-stage CCA has been used in many countries including Austria, Australia, Canada, Ireland, the Netherlands, Slovakia, Switzerland, and the UK (Mochon and Saez, 2017; Cave and Nicholls, 2017; Bichler and Goeree, 2017).

One takeaway message from the recent literature is that the design of spectrum auctions is still a topic of intense debate. Another is that it requires different approaches – theory, laboratory experiments, and simulations – to understand the properties of alternative formats. Mechanism design theory has identified the unique efficient auction in which bidding truthfully is a (weakly) dominant strategy so that bidders do not require information about their rivals' valuations. Despite its desirable features, the Vickrey-Clarke-Groves (VCG) mechanism is rarely used in the field for various practical reasons (Ausubel and Milgrom, 2006). Bayesian-Nash implementation allows for a broader class of auction formats but imposes a strong common-prior assumption. Moreover, recent game-theoretical models of spectrum auction formats typically make simplifying assumptions about bidders' valuations (Goeree and Lien, 2014), focus on small environments with a few items and players (Levin and Skrzypacz, 2016), or assume complete information to highlight strategic problems (Janssen and Karamychev, 2016). Laboratory experiments provide valuable insights as well, e.g. Goeree and Holt (2010), but the size of the markets that can be organized in an economic experiment is typically limited. And, like in theoretical analyses, experimental designs often ignore or simplify complicated institutional details (e.g. activity rules or spectrum caps).

In contrast, simulations allow one to analyze realistic market sizes and to take institutional details into account. As such they can provide complementary insights (Consiglio and Russino, 2007). Interestingly, there are no published simulation studies about spectrum auction markets that we are aware of, even though simulations are regularly used by consultants and telecoms to explore different bidding strategies.

1.1. Auctions as Algorithms

Spectrum auctions can be seen as large games with many bidders, licenses, and additional rules such as spectrum caps and activity rules. For example, the 2014 Canadian 700MHz auction allowed bidders to bid on 18 packages in 14 regions leading to 18^{14} possible packages. In large games like this, bidders need a lot of information about competitors to bid strategically, and one might argue that strategic manipulation is less of a concern. But even if we ignore strategic bidding, it is far from obvious that auctions yield efficient outcomes. It is well-known that the allocation problem in a combinatorial auction where bidders have preferences for combinations of licenses is an NP-hard optimization problem (Cramton et al., 2006). The SMRA and different versions of the CCA can be interpreted as algorithms to solve this problem, and it is important to understand the approximation ratios of these algorithms (Domowitz and Wang, 1994). It is unclear whether to expect efficient outcomes even when bidders bid straightforwardly in each round of the auction.

Theoretical results on the allocation problem in combinatorial auctions are not encouraging. There is no polynomial-time algorithm that guarantees an approximate solution to the winner determination problem within a factor of $l^{1-\varepsilon}$ from the optimal allocation, where l is the number of submitted bids and ε a small number (Pekec and Rothkopf, 2003). The problem is APX-hard and the worst-case approximation ratio of any polynomial-time algorithm for the allocation problem in combinatorial auctions is in $O(\sqrt{m})$, where m is the number of objects to be sold. In large spectrum auctions with many licenses such as the Canadian auction in 2014, this lower bound on efficiency is obviously very low[1] and provides no practical guidance. There are also results on the worst-case efficiency of the single-stage CCA with bidders who truthfully reveal their preferences (i.e., bid straightforward) in each round. This can also be seen as an algorithm to solve the allocation problem. Unfortunately, the worst-case approximation ratio can be $2/(m+1)$ with m being the number of licenses (Bichler et al., 2013b).

Worst-case bounds might be too pessimistic, and it is interesting to understand the average-case approximation ratio to the fully efficient solution of different auction types based on realistic problem instances. Numerical experiments are widely used in operations research and computer science to analyze the average-case solution quality of an algorithm, and they help to understand aspects of the algorithm that do not lend themselves to theoretical analysis or lab experiments. In particular, we want to study the average-case efficiency of different auction algorithms under the assumption of straightforward bidding. This provides an estimate of the efficiency that can be achieved by the auction algorithms that are commonly employed in the field *for realistic market sizes and considering all details of the auction design.*

Another important element of realism in our simulations stems from the fact that we estimate bidder value models from bids observed in the Canadian 700MHz auction. Unlike other regulators, the Canadian regulator, Industry Canada, revealed detailed bid data.[2] We use the supplementary bids from the Canadian 700MHz auction to estimate individual license values for each bidder.[3] Our goal is not to provide the most precise estimates of bidders' private valuations and then reproduce the outcomes of the Canadian auction. This would lead to over-fitting problems in the counterfactuals we have in mind. Our goal instead is to robustly infer reasonable instances of bidder valuations and then compare the performance of different auction formats under various "out-of-sample" synergy conditions.

In particular, we compare efficiency of the SMRA to that of the single-stage and two-stage CCA using extensive numerical experiments. The experiments employ a simulation framework that implements the exact rules of the 2014 Canadian auction, i.e. band plan, spectrum caps, regional interests of bidders and rules of the two-stage CCA. In addition, we implemented the rules of SMRA and the single-stage CCA as they were used in other countries. Since the Canadian auction shares similarities with other large spectrum auctions, e.g., in Australia, India, and the US, the results of our study are of interest beyond the Canadian market.

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[1] $\sqrt{18 * 14} = 15.87$, such that the worst-case approximation of any polynomial-time algorithm might be $OPT/15.87$, where OPT is the optimal solution to the allocation problem.

[2] Besides Industry Canada, the UK's Ofcom is the only regulator that disclosed bids data for their CCA. However, the auctions in the UK have only national licenses and there are too few bidders and objects to conduct a simulation study as in this paper.

[3] The bid data used in this paper are available for replication studies.

1.2. Outline

The paper is organized as follows. The next section provides details about the SMRA and CCA formats. Section 3 covers the experimental design, including details about the Canadian 700MHz auction. The results of the simulation experiments can be found in Section 4. Section 5 concludes and the Appendices provide further details about the value model we estimate (Appendix A), the optimization algorithms used in the simulations (Appendix B), our modeling assumptions (Appendix C), and the results (Appendix D).

2. The Auctions

In this section, we briefly summarize the SMRA and different versions of combinatorial clock auctions and provide references to the details of the implementations, which follow the very auction rules used in Canada or other countries.

2.1. The Simultaneous Multiple Round Auction (SMRA)

The SMRA is an extension of the English auction to more than one license. All licenses are sold at the same time, each with a price associated with it, and the bidders can bid on any one of the licenses. The auction proceeds in rounds, which is a specific period of time in which all bidders can submit bids. After the round is closed, the auctioneer discloses who is winning and the prices of each license, which coincide with the highest bid submitted on each license. There are differences in the level of information revealed about other bidders' bids. Sometimes all bids are revealed after each round, sometimes only prices of the currently winning bids are published.

The bidding continues until no bidder is willing to raise the bid on any of the licenses any more. In other words, if in one round no new bids are placed, the bidders receive the spectrum for which they hold the highest active bid, then the auction ends with each bidder winning the licenses on which he has the high bid, and paying its bid for any license won.

SMRA uses simple *activity rules* which enforce bidder activity throughout the auction. Monotonicity rules are regularly used, where bidders cannot bid on more licenses in later rounds. This forces bidders to be active from the start. Typically, bidders get eligibility points assigned at the start of the auction, which define the number of licenses they are allowed to bid on maximally. If the number of licenses they win in a round and the new bids they submit require less eligibility points than in the last round, then they risk losing points, which limits the number of items they can bid on in future rounds.

Apart from the activity rules, there are typically additional rules that matter. Auctioneers set *reserve prices* for each license, which describe prices below which an license will not be sold. They need to define *bid increments* and how bid increments might change throughout the auction. A bid increment is the minimum amount by which a bidder needs to increase his bid beyond the ask price in the next round. Sometimes, auctioneers allow for *bid withdrawals* and sometimes bidders get bid waivers, which allow bidders not to bid in a round without losing eligibility points. Finally, auctioneers often set *bidding floors and caps*, which are limits on how much a winner in the auction needs to win at a minimum and how much he can win at most. These rules should avoid unwanted outcomes such as a monopoly after the auction or a winner who wins so little spectrum that it is not sufficient for a viable business.

The auction format is popular because it is easy to implement and the rules are simple. If the valuations of all bidders were additive, the properties of a single-object ascending auction carry over. Unfortunately, this is rarely the case and bidders have often synergies for specific licenses in a package or their preferences are substitutes. Only if bidders have substitutes preferences and bid straightforwardly, then the SMRA terminates at a Walrasian equilibrium, i.e., an equilibrium with linear prices (Milgrom, 2000). *Straightforward bidding* means that a bidder bids on the bundles of licenses, which together maximize the payoff at the current ask prices in each round. Milgrom (2000) also showed that with at least three bidders and at least one non-substitutes valuation (for example super-additive valuations for a package if licenses) no Walrasian equilibrium exists.

We assume a simple straightforward bidding strategy in each round where bidders submit bids on their payoff maximizing package. The exposure problem is a key strategic challenge in the SMRA. In our simulations we assume that bidders take different levels of exposure. Either they only bid up to the additive values of a package and ignore the synergies or they exceed the additive value of the licenses in a package and take some exposure risk. This is a treatment variable in the experiments. More details can be found in Section 3.5 and Appendix C.

2.2. Alternative Versions of the Combinatorial Clock Auction

There are various versions of combinatorial clock auctions that have been implemented in the field: the *two-stage combinatorial clock auction (CCA)* that was used in e.g. the Canadian 700MHz auction, the *two-stage combinatorial clock auction with base and OR bids in the second stage (CCA+)* as it was used in the Canadian 2.5GHz spectrum auction, and the *single-stage combinatorial clock auction (SCCA)*, which was recently used in Romania and Denmark.

2.2.1. The Two-Stage Combinatorial Clock Auction (CCA)

The two-stage combinatorial clock auction was introduced by Ausubel et al. (2006). In contrast to SMRA, the auction avoids the exposure problem by allowing for bundle bids. Maldoom (2007) describes a version as it has been used in spectrum auctions across Europe. In a two-stage combinatorial clock auction, bids for bundles of licenses are made throughout a number of sequential, open rounds (the primary bid rounds or clock phase) and then a final sealed-bid round (the supplementary bids round). In the primary bid rounds the auctioneer announces prices and bidders state their demand at the current price levels. Prices of licences with excess demand are increased by a bid increment until there is no excess demand anymore. Jump bidding is not possible. In the primary bid rounds, bidders can only submit a bid on one bundle per round. This rule is different to the initial proposal by Ausubel et al. (2006). If bidders bid straightforward on their payoff maximizing bundle in each round and all goods get sold after the clock phase, allocation and prices would be in competitive equilibrium. It might well be that there is excess supply after the clock phase, however. The sealed-bid supplementary bids phase and a Vickrey-closest core-selecting payment rule try to induce truthful bidding and avoid incentives for demand reduction. This is because core payments in Day and Cramton (2012) are computed such that a losing bid of a winner does not increase his payment for his winning bid. The winner determination after the supplementary bids round considers all bids, which have been submitted in the primary bid rounds and the supplementary bids round and selects the revenue maximizing allocation. The bids by a single bidder are mutually exclusive (i.e., the CCA uses an XOR bidding language).

Activity rules should provide incentives for bidders to reveal their preferences truthfully and bid straightforwardly already in the primary bid rounds. Bidders should not be able to shade their bids and then provide large jump bids in the supplementary bids round. An eligibility-points rule is used to determine activity and eligibility to bid in the primary bid rounds. Each license in a band requires a certain number of eligibility points, and a bidder cannot increase his activity across rounds. In the supplementary bids round, revealed preferences during the primary bid rounds are used to derive relative caps on the supplementary bids that impose consistency of preferences between the primary and supplementary bids submitted. The consequence of these rules is that all bids are constrained relative to the bid for the final primary package by a difference determined by the primary bids. This should set incentives for straightforward bidding in the primary bid rounds.

2.2.2. The Two-Stage Combinatorial Clock Auction with OR Bids (CCA+)

The two-stage combinatorial clock auction reaches full efficiency if bidders bid on their payoff maximizing package during the primary phase and truthfully on *all* packages in the supplementary phase. The number of packages increases exponentially in the number of items and in the Canadian auction this would not be possible for national bidders. To counter this "missing bids" problem (see Bichler et al. (2013a)), Industry Canada introduced the possibility to submit one or more mutually exclusive collections of OR bids[4] in the supplementary round, in addition to the mutually exclusive XOR bids. These OR bid collections (see Footnote 4) are used in conjunction with the bidder's final primary package and can be used to express

[4] `http://www.ic.gc.ca/eic/site/smt-gst.nsf/eng/sf10730.html#aD-s10`

additive valuations on top of the final primary package bid in a concise manner.

2.2.3. The Single-Stage Combinatorial Clock Auction (SCCA)

We also analyze the single-stage combinatorial clock auction as it has been described by Porter et al. (2003). This format was used in Romania[5] in 2012, and in the recent the Danish 1800Mhz auction[6] in 2016. Here, bidders can place a bid on one or multiple package bids at the current ask prices in each round. The pricing rule is simple: As long as at least one item is over-demanded, the prices for these over-demanded items increase by a bid increment. If at some point the supply equals demand, the auction terminates and assigns the items to all bidders. In the case of excess supply, the auctioneer considers all bids, including those of the previous rounds, and solves a winner determination problem. If the all winning bidders in the current round are in the winner determination problem's solution, the auction terminates. Otherwise, a new round begins with increased prices on all items that were not allocated in the previous round. Note that the Danish 1800MHz auction had some additional rules about the prices one can specify for package bids in each round that were not considered in our simulations. We argue that these differences do not influence the results of the research question in this paper significantly.

3. Experimental design

Our experiments were conducted by using an auction framework which allows the run of all of the major spectrum auction formats, i.e. the SMRA, the single-stage and two-stage CCA (SCCA, CCA), and the two-stage CCA with OR bids (CCA+). The implementation follows the very rules specified in the documents that we referenced in the previous sections. In addition, we used the exact band plan, the licenses and caps of the Canadian 700MHz auction. Next, we introduce the value model and the strategies of the automated bidders, before we summarize the experimental design.

3.1. Market Environment

We will briefly summarize the environment of the Canadian 700 MHz auction in 2014. We used the very same band plan, the same start prices, and the same spectrum caps as in this auction.[7]

[5] http://www.ancom.org.ro/en/uploads/forms_files/terms_of_reference1331893175.pdf

[6] https://ens.dk/sites/ens.dk/files/Tele/information_memorandum_june_2016.pdf

[7] The detailed auction rules of the 2014 Canadian 700Mhz auction can be found at http://www.ic.gc.ca/eic/site/smt-gst.nsf/eng/h_sf01714.html.

Licenses

The band plan consists of five paired spectrum licenses (A, B, C, C1, and C2), and two unpaired licenses (D, E) in 14 service areas. B and C as well as C1 and C2 were treated as generic licenses, i.e., substitutes. Although the licenses are all in the 700MHz band, they are technically not similar enough to sell all of them as generic licenses of one type.

Market Participants

The auction was dominated by three national carriers Bell, Rogers, and Telus. Rogers was the strongest bidder and contributed 62.45% to the overall revenue, while Telus paid 21.69% and Bell 10.73%. Rogers did not bid on C1/C2 and aimed for licenses in A, and B/C throughout the auction, while Bell and Telus also bid on C1/C2 in certain service areas. The smaller bidders mainly bid on remaining C1/C2 licenses. Bell and Telus had to coordinate and find an allocation such that they both got sufficient coverage in the lower 700 MHz band (A, B and C licenses), which explains much of the bid data. The bid data generated for our experiments was based on the field data.

Caps

In order to facilitate the market entry for new entrants, Industry Canada set up spectrum caps. All eight bidders were restricted to at most 2 paired frequency licenses in each service area. Large national wireless service providers such as Rogers, Bell, and Telus were further limited in that they could only bid on one paired license in each service area among licenses B, C, C1 and C2. This cap on large wireless service providers did not, however, include license A. Still, the national bidders could bid on $2 * 3 * 3 = 18$ packages per region including the empty package, which leads to $18^{14} \approx 3.75 * 10^{17}$ packages across all regions. In contrast, in the Canadian 700MHz auction, Rogers submitted 12 supplementary bids, Bell 543 and Telus 547 bids.

Activity Rules

All implemented iterative auction formats use an eligibility point or, in all CCA formats, a revealed preference/eligibility point activity rule during the rounds. Bidders begin each round with a number of eligibility points and they can only bid on a set of licenses that in sum requires a less or equal number of eligibility points. The rounds' activity requirement is 100%, i.e., a bidder loses all eligibility points that he does not use in a round. Industry Canada allowed bidders to bid on packages even beyond the current eligibility point limit, as long as this choice is consistent with the bidder's revealed preferences up to this point.

The two-stage CCA formats apply this rule also to all supplementary bids: All bids submitted in this stage have to be consistent with the bidder's revealed preferences, taking into account his bid in the final clock round and all eligibility-reducing rounds starting from the last round in which the bidder's eligibility was at least as high as the total points associated with the current bid.[8]

3.2. Estimates of Base Valuations

In order get reasonable problem instances, we constructed bidder-specific value models for all participants from the auction data. The Canadian regulator published all bids from the 700Mhz spectrum auction, which allowed us to estimate realistic valuations. We do *not* aim to get precise value estimates with the goal to reproduce the outcomes of the Canadian 700MHz auction. Rather we aim to obtain realistic valuations from the observed package bids in the supplementary stage of the Canadian auction which have a reasonable order of magnitude with respect to the starting and end prices in the auction. For this, we fitted a L1 linear regression to the supplementary bids. As an abstract example, suppose there are three items, A, B, and C, and a bidder submits the following package bids: $b(AB) = 10$, $b(AC) = 10$, $b(BC) = 10$, and $b(ABC) = X$. Then an L1 regression that explains these bids in terms of underlying license values yields $v_A = v_B = v_C = 5$ *irrespective* of X.[9] The estimated valuations of course depend on the estimation technique, but our main simulation results are robust and hold for very different samples.

Interestingly, we find that the linear model fits the data well (see A for details), which suggests that the regression results provide reasonable estimates of the bidders' license values. As in the abstract example, there were some non-negligible residuals for larger packages but this is to be expected given the synergistic nature of bidders' values. We purposefully do not try to estimate these package synergies. First, this would be difficult given that only a small proportion from the exponential set of possible package bids were submitted.[10] More importantly, we are not interested in reproducing the outcomes of one particular auction, i.e. the Canadian 700MHz, but want to robustly compare the performance of the SMRA and CCA under various "out-of-sample" synergy conditions.

3.3. Synergy Model

We will consider two synergy models: with "linear" synergies the marginal value of each license rises linearly with the total number of licenses won. In contrast, with "extreme national synergies" the marginal value of a license is independent of

[8] Details of the activity rule can also be found at `http://www.ic.gc.ca/eic/site/smt-gst.nsf/eng/h_sf01714.html`.

[9] An L2 regression would instead yield $v_A = v_B = v_C = (X + 20)/7$.

[10] Telus submitted the most supplementary bids: 547. But this is only 0.00000000000015% of all possible package bids.

the number of licenses won unless *all* licenses in a national package are won. To illustrate, consider an abstract example with K items that all have one eligibility point. Denote a bidder's individual license values by ϑ_k for $k = 1, \ldots, K$. When there are synergies, the individual license values go up when they are part of a larger package. Suppose license k is part of a set of L licenses that the bidder wins then

$$\vartheta_k(L) \;=\; \vartheta_k\left(1 + (\alpha - 1)\frac{L}{K}\right) \tag{1}$$

where the synergy coefficient, $\alpha \geq 0$, determines the scale of package valuations. We are mainly interested in the case where licenses are complements ($\alpha > 1$), but the value parametrization in (1) also applies to the substitutes case ($\alpha < 1$). When $\alpha = 1$, there are no synergies (neither positive nor negative) and values are simply additive.

With the "linear synergy" model that license values rise linearly with the number of licenses won. In the "extreme national synergy" model, non-additive valuations are limited to

$$\vartheta_k(L) \;=\; \begin{cases} \vartheta_k & \text{if} \;\; 1 \leq L < K \\ \alpha\vartheta_k & \text{if} \;\; L = K \end{cases}$$

With extreme national synergies the marginal value of each license is independent of the number of licenses won unless *all* licenses in the national package are won, see also Goeree and Lien (2014). The fact that synergies are only applied for a selected few out of all possible packages is an extreme case. In the field we have seen similar models where super-additive valuations were only determined for a small number of packages as it is often difficult to determine the right polynomial. However, there is no public information about the structure of such valuation models and the level of synergies that we are aware of. We will see that with such a synergy model, package auctions do achieve higher efficiency, but this is not the case with the linear synergy value model.

The experiments use the following parameters: $\alpha \in \{1, 2, 2.5\}$ and both synergy models. A difference with the abstract example above is that in the Canadian 700MHz auction not all licenses had the same eligibility points, but it in the case of the linear model it is straightforward to account for those.[11] Another difference is that in the Canadian auction there were "national bidders", i.e. Telus, Bell, and Rogers, and "regional bidders". In the experiments we assume that only the national carriers enjoy synergies. A final detail is that national bidders could cover the country once or twice with a specific set of paired licenses: In the case of covering the country twice, the synergy coefficient for the national package was α, whereas if a

[11]Let e_k denote the eligibility points associated with license $k = 1, \ldots, K$. Suppose license k is part of a winning set S. Define $e = \sum_{k=1}^{K} e_k$ and $e_S = \sum_{k \in S} e_k$ then $\vartheta_k(L) \;=\; \vartheta_k\Big(1 + (\alpha - 1)\frac{e_S}{e}\Big)$.

bidder covers the nation within a specific paired frequency block once[12], his synergy coefficient is $\alpha - 0.5$.

3.4. Efficient Allocation

Given the large number of licenses and bidders the computation of the optimal allocation is challenging. Obviously, we cannot enumerate all valuations for all possible packages of all bidders. Instead, we solve a mixed integer program, which leverages the structure of the valuation models described above without having to enumerate all possible packages. The model is described in Appendix B.[13] A version of this model is also used to compute the payoff-maximizing package in each round for bidders.

3.5. Bidding Strategies

There are no closed-form equilibrium bidding strategies for the SMRA or the CCA in complex environments such as the Canadian 700MHz auction. But strategic manipulation would be difficult given the large number of licenses and packages bidders are interested in, which is why we assume bidders naively optimize in each round of the auction. Such *straightforward bidding* means that a bidder bids on the package with the highest net value taking into account current prices. Formally, a straightforward bid $\beta_j^{\mathrm{SF}}(v_j, p_j)$ is defined as

$$\beta_j^{\mathrm{SF}}(v_j, p_j) \in \arg\max_{S \subseteq I}(v_j(S) - p_j(S))$$

where S is the set of items bidder j wants to bid on from all possible items I, $v_j(S)$ is the bidder's valuation for the package S and $p_j(S)$ the price he has to pay for it. In the case of an exposure limit λ, we limit the value $v_j(S)$ so that $v_j^{\lambda}(S) = min\left(v_j(S), \lambda \sum_{i \in S} v_j(i)\right)$ for all S where $p_j(S) > \sum_{i \in S} v_j(i)$.[14]

3.6. Experimental Design

The main treatment variables in our experiments are the auction format, the synergies in the two value models (α_l or α_n), and the exposure risk that bidders are willing to take in SMRA. We also consider limits on the number of bids that bidders are willing to submit.

[12]As an example, winning one licence in each region in the A frequency block would cause a higher surplus, whereas covering the nation once with a mix of A and BC licences would not.

[13]The optimization model is an effective way to determine payoff-maximizing packages in each round with both synergy models. Unfortunately, with non-linear, e.g. quadratic, synergies the integer programming problem becomes intractable.

[14]As an example, assume a bidder with $v(i_1) = 5, v(i_2) = 5, v(i_3) = 20$ and $v(\{i_1, i_2\}) = 100$. Assume further a fixed number $5 < \rho < 10$, $p(i_1) = p(i_2) = p$ and $p(i_3) = 2\rho$, and a winning cap of two items per bidder. With $\lambda < 2$, β_j^{SF} is $\{i_3\}$, whereas with a $\lambda > 2$, the bidder strictly prefers the package $\{i_1, i_2\}$.

Variables	Values
Auction formats	SMRA, SCCA(1), SCCA(2), CCA(0), CCA(200), CCA+(200)
Synergy model	No, National-2.0, National-2.5, Linear-2.0, Linear-2.5
Exposure	No, +50%, +100%, Full

Table 1: Treatment variables

We compare SMRA against various versions of the combinatorial clock auction format, which include the single stage CCA where bidders submit one or two bids per round, SCCA(1) and SCCA(2), the two stage CCA with zero[15] or 200 additional bids, CCA(0), CCA(200). We also analyze the CCA with an OR bid language in the supplementary stage, in addition to 200 supplementary bids, CCA+(200).

We draw 100 sets of random valuations based on the derived value models from a uniform distribution of ±5% around the item valuations and subsequently use these valuations for all treatment combinations, which are described in Table 1. In total, we have $9*4+6 = 42$ treatment combinations and 4,200 simulation runs with approx. 1,500 hours of simulation run time.

We use efficiency $E(X)$ as the primary aggregate measure.[16] Let the optimal allocation be denoted X^* then:

$$E(X) = \frac{\sum_{j\in J} v_j(X)}{\sum_{j\in J} v_j(X^*)}$$

We also measure the revenue distribution $R(X)$, which compares the auctioneers revenue against the optimally achievable surplus.

$$R(X) = \frac{\sum_{j\in J} p_j(X)}{\sum_{j\in J} v_j(X^*)}$$

4. Results

In the following, we summarize the main results of the numerical experiments. The efficiency of the auction formats is quite high and with a synergy value of less or equal to 2, efficiency is typically higher than 80% of the optimal surplus. This is surprising given that the auctions are simple heuristics and the worst-case approximation ratio for the allocation problem are low, as we discussed in the introduction.

[15] This means that bidders submit only supplementary bids on combinations they already bid for in the clock phase. The result of the clock phase of the two stage CCA is also added for comparison.

[16] A formulation of the mixed integer program can be found in Appendix B.

Result 1. *The two synergy models lead to significantly different results:*

- *With linear synergies, the efficiency level of the SMRA exceeds those of the various CCA versions. This is true even for a high value of the synergy coefficient α_l. Among the CCA versions, the two stage CCA is more efficient than the single stage SCCA.*

- *With the extreme national synergies, the single- and two-stage CCA yield comparable efficiency levels, which exceed those of the SMRA. CCA+ has a significantly higher effiency than all other formats. Efficiency in the SMRA and in the CCA formats decreases for higher levels α_n, but not for the CCA+.*

For the national synergy model, efficiency levels are ranked as follows: CCA+ $\succ^{***}$ SCCA $\succ^{**}$ CCA $\succ^{***}$ CCA (clock) $\succ^{***}$ SMRA. For the linear synergy model, the ranking is: SMRA $\succ^{***}$ CCA+ $\sim$ CCA $\succ^{***}$ SCCA $\succ^{***}$ CCA (clock).

We provide box plots with the efficiency levels in the appendix in Figures 1 to 10, which show the results for different levels of synergy. Figure 1 shows box plots in a market with purely additive valuations where the efficiency of all auction formats is very high. SMRA achieves full efficiency while the combinatorial clock auction formats have a slightly lower efficiency with a median above 97%. This efficiency loss might be due to the missing bids problem, as bidders can only reveal a small subset of all their bundle preferences in a combinatorial auction of this size. SMRA_{No} refers to an implementation of SMRA with bidding agents who do not take any exposure risk, while they bid up to their full package valuation in SMRA_{Full}.

Figure 2 shows the box plots for the extreme national synergy model with a synergy coefficient of $\alpha_n = 2.0$ while Figure 3 shows the box plots for $\alpha_n = 2.5$. With national synergies, the median efficiency of the SMRA with bidders who do not take exposure risk (SMRA_{No}) is 0.77 or 0.69 respectively. If bidders would take full exposure, some of the efficiency would be recovered. The efficiency of the CCA formats is high at around 0.96 and 0.85 for the CCA+ and CCA, respectively.

Figures 4 and 5 show efficiency for the linear synergy model with synergy coefficients of $\alpha_l = 2.0$ and $\alpha_l = 2.5$. Interestingly, efficiency of SMRA remains very high for both synergy levels. An explanation for this is the auction format and the caps in the Canadian auction. The caps limited the size of the packages any national bidder could win. None of the bidders could win all a huge package with all licenses. All national bidders were also able to win larger packages in SMRA, which all result in some level of complementarity in the linear synergy model. The average efficiency of the combinatorial clock auctions is substantially lower.

The pattern in the box plots is reflected in the regression analysis. Table 2 summarizes the OLS estimates for the extreme national synergy model and Table 3 does the same for the linear synergy model (the dependent variable in each case is efficiency). SMRA describes the baseline auction format in this regression and no exposure the baseline for the various exposure levels that the bidders take. In Table 2

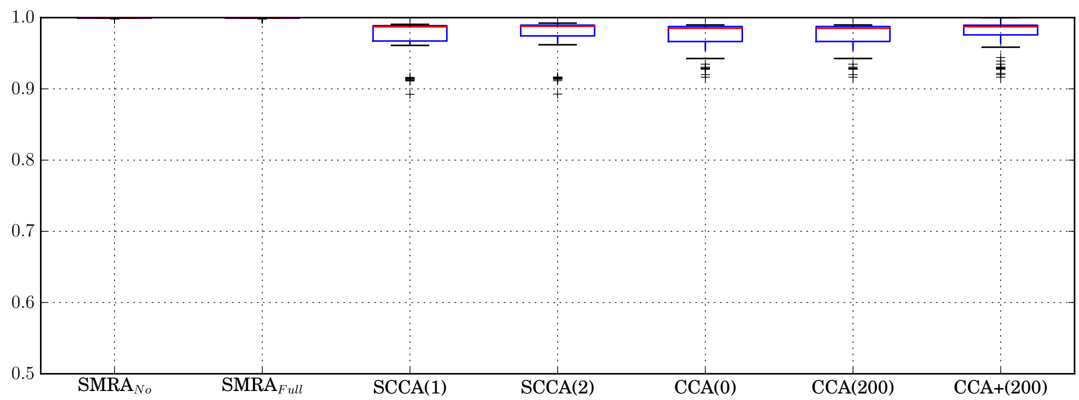

Figure 1: Efficiency in the additive value model

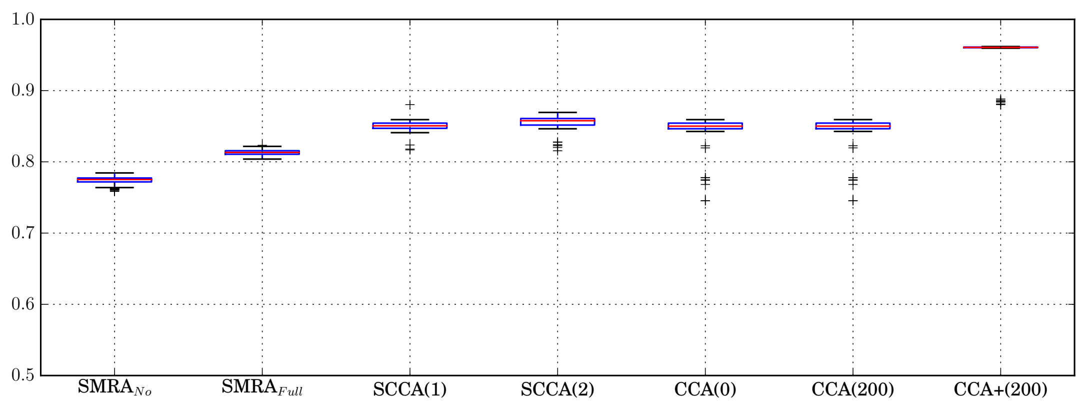

Figure 2: Efficiency in the National-2.0 value model

all combinatorial clock auction formats have a positive and significant influence on efficiency compared to SMRA. Differences among the combinatorial clock auction formats are small. In contrast, in Table 2 all the regression coefficients are negative indicating a negative impact of alternative auction formats on efficiency compared to SMRA. The CCA auction with the OR bid language (CCA+) was best among the combinatorial clock auction format as bidders revealed a significantly larger number of package valuations.

One of the specifics of the Canadian value model is the number of three national competitors. We ran also simulations with four national bidders[17], but the main results regarding the comparison of auction formats remain.

[17] A fourth national bidder was created by first averaging the estimated valuations of the three national bidders. We then drew 100 sets of random valuations based on this model from a uniform distribution of ±5%, identically to the procedure used for our main simulations.

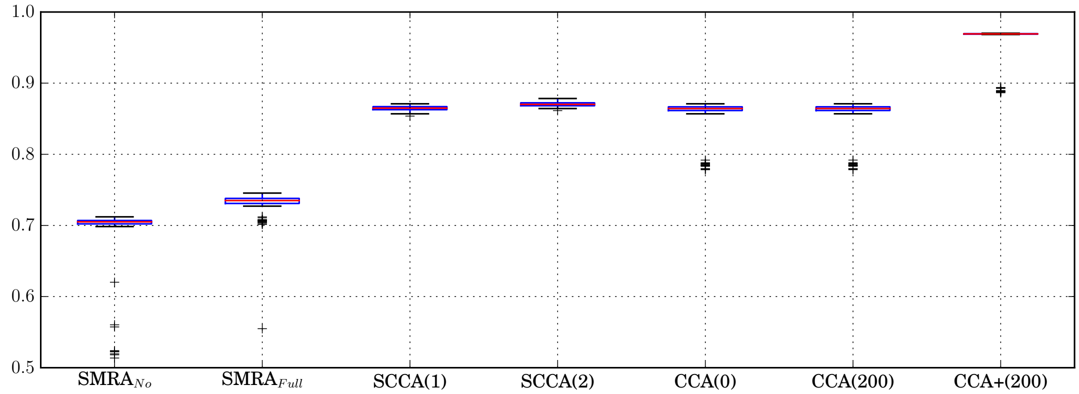

Figure 3: Efficiency in the National-2.5 value model

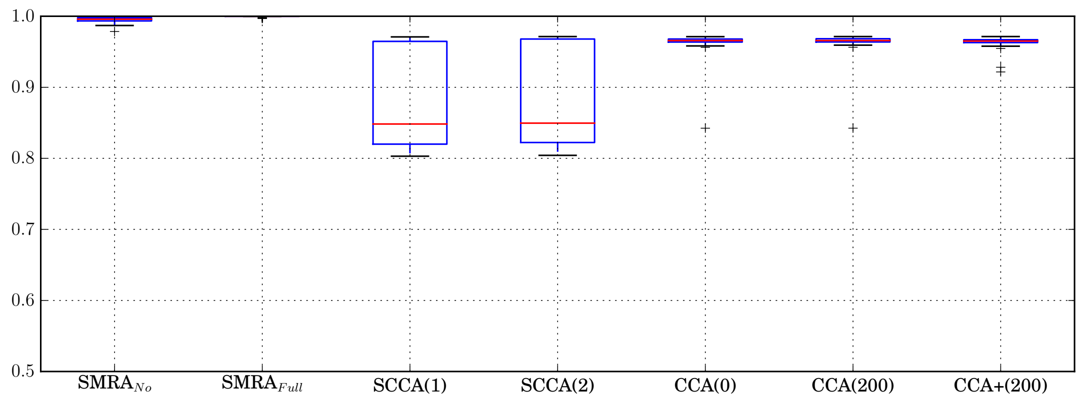

Figure 4: Efficiency in the Linear-2.0 value model

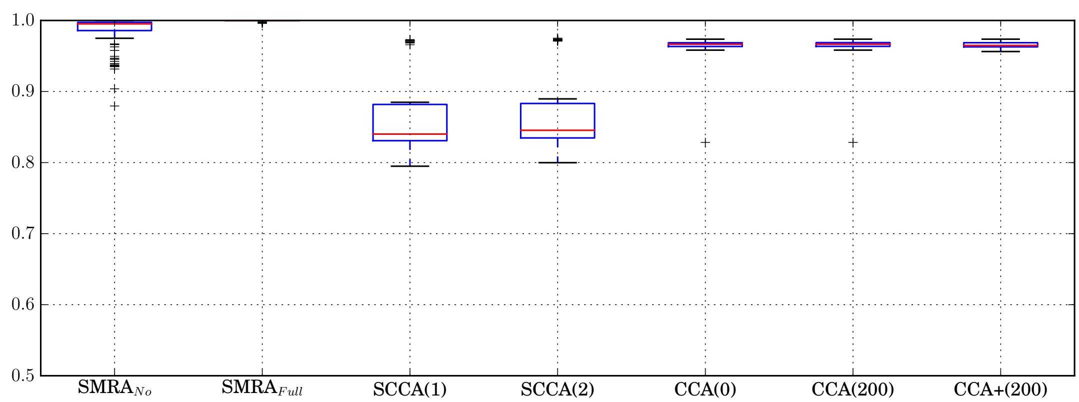

Figure 5: Efficiency in the Linear-2.5 value model

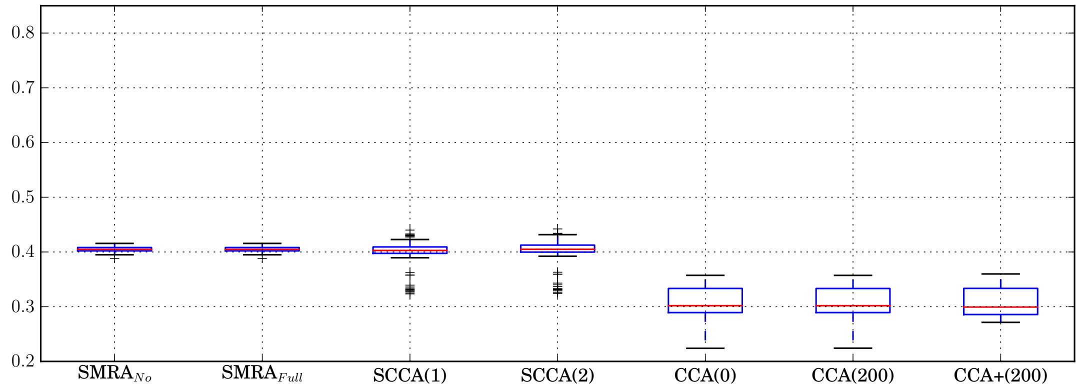

Figure 6: Revenue in the additive value model

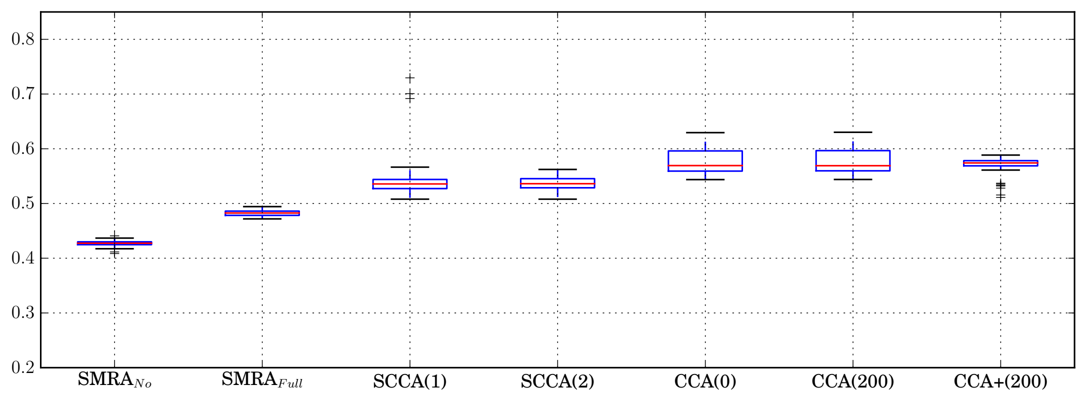

Figure 7: Revenue in the National-2.0 value model

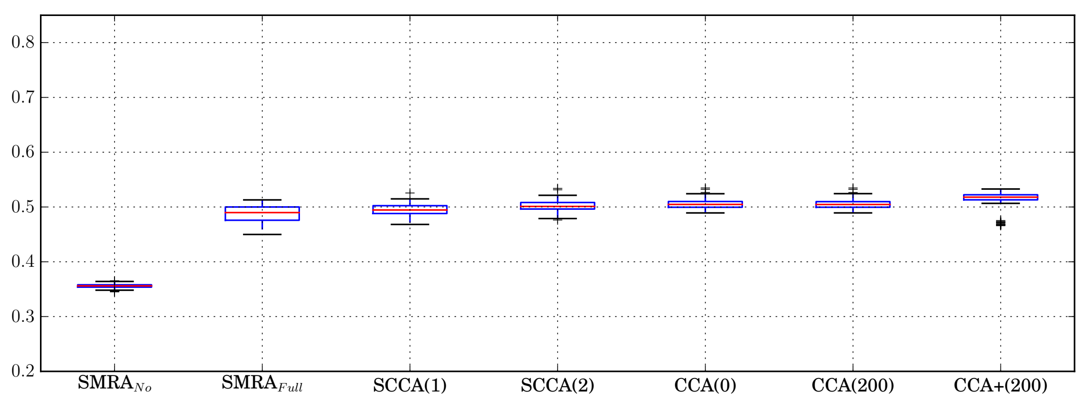

Figure 8: Revenue in the National-2.5 value model

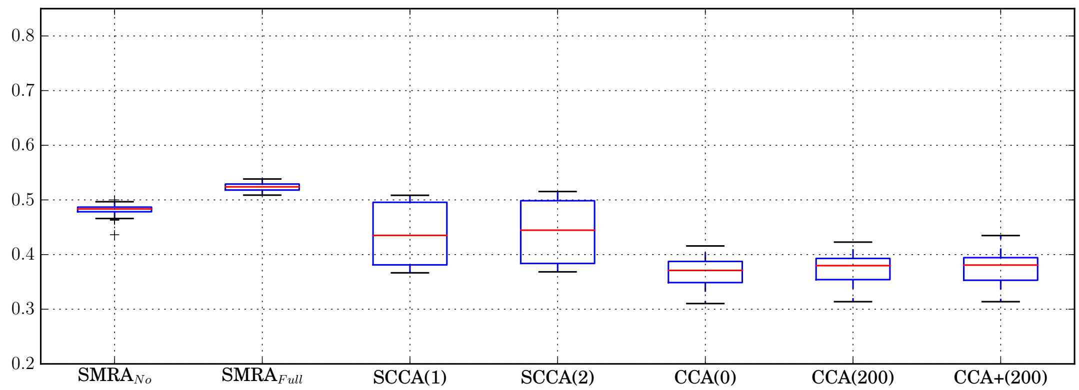

Figure 9: Revenue in the Linear-2.0 value model

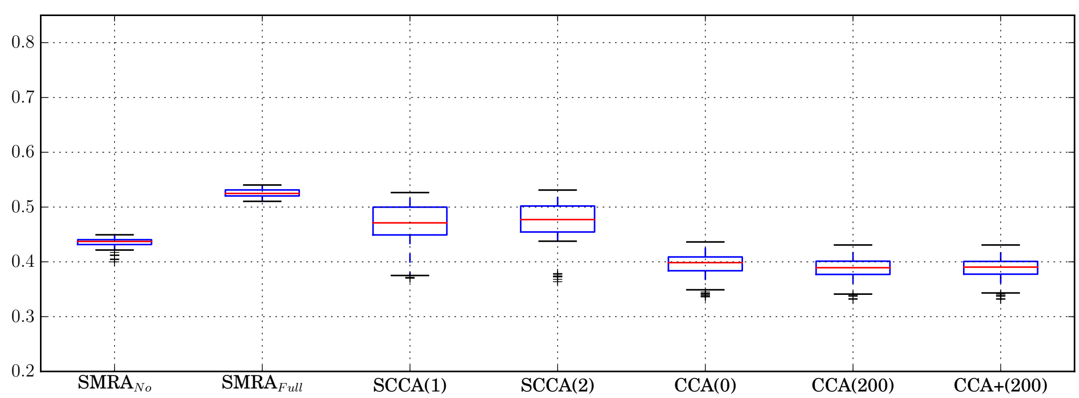

Figure 10: Revenue in the Linear-2.5 value model

Coefficients	$E(X)$	p-value	$R(X)$	p-value
Intercept	0.8568	< 0.001	0.5965	< 0.001
Auction Format				
CCA (clock)	0.1050	< 0.001	0.0989	< 0.001
CCA(0)	0.1176	< 0.001	0.1497	< 0.001
CCA(200)	0.1177	< 0.001	0.1499	< 0.001
CCA+(200)	0.2253	< 0.001	0.1505	< 0.001
SCCA(1)	0.1256	< 0.001	0.1266	< 0.001
SCCA(2)	0.1310	< 0.001	0.1282	< 0.001
Synergy	-0.0556	< 0.001	-0.0911	< 0.001
Exposure				
+50%	0.0344	< 0.001	0.0506	< 0.001
+100%	0.0395	< 0.001	0.0933	< 0.001
Full	0.0390	< 0.001	0.0931	< 0.001
Adjusted R^2	0.79		0.84	

Table 2: Regression results for the extreme national synergy model (base: SMRA_{No}, no exposure) with efficiency and revenue as dependent variables.

Coefficients	$E(X)$	p-value	$R(X)$	p-value
Intercept	1.0125	< 0.001	0.4941	< 0.001
Auction Format				
CCA (clock)	-0.1425	< 0.001	-0.0796	< 0.001
CCA(0)	-0.0246	< 0.001	-0.0784	< 0.001
CCA(200)	-0.0244	< 0.001	-0.0781	< 0.001
CCA+(200)	-0.0243	< 0.001	-0.0784	< 0.001
SCCA(1)	-0.1185	< 0.001	-0.0037	-
SCCA(2)	-0.1158	< 0.001	-0.0006	-
Synergy	-0.0104	< 0.001	-0.0159	< 0.001
Exposure				
+50%	0.0062	< 0.1	0.0030	-
+100%	0.0084	< 0.05	0.0272	< 0.001
Full	0.0106	< 0.01	0.0663	< 0.001
Adjusted R^2	0.74		0.66	

Table 3: Regression results for the linear synergy model (base: SMRA_{No}, no exposure) for efficiency and revenue as dependent variables.

Result 2. *Allowing OR bids in the the supplementary phase of the two-stage CCA significantly raises efficiency and revenue compared to the CCA with XOR bid language (p-value:* < 0.001*) in the extreme national synergy model, but not in the linear synergy model. Bidding up to 200 additional supplementary bids does not significantly improve efficiency compared to a two-stage CCA without additional package bids.*

Straightforward bidding is fully efficient in the two stage CCA if bidders are able to provide their preference for all packages. In larger combinatorial auctions such as the Canadian auction, this is not possible with a fully combinatorial XOR bid language. Bidding only on a small fraction of the 10^{17} possible packages in the supplementary phase does not improve the allocation significantly, even if bidders bid on up to 200 of their most valuable packages. The addition of OR bids in the supplementary phase does improve the efficiency of the auction outcome, but this can only be observed in the extreme national synergy model.

Result 3. *With national synergies, revenue is lowest in the SMRA when bidders do not take any exposure. With linear synergies, revenue of the two-stage CCA (and CCA+) is lowest. The single-stage CCA achieves higher revenue than the two-stage CCA on average in the linear synergy model.*

Figure 6 shows the revenue results of the additive model, where the two-stage CCA, both CCA(0) and CCA(200), is worst. Figures 7 to 8 provide box plots for the extreme national synergy model, while Figures 9 to 10 shows those for the linear synergy model. Since efficiency is low with extreme national synergies, also revenue is low. The opposite is true for linear synergies. The revenue of the single-stage CCA, SCCA(1) and SCCA(2), is always higher on average than that of the two-stage CCA, CCA(0) and CCA(200).

Table 2 summarizes the robust regression estimates for the extreme national synergy model and Table 3 shows those for the linear synergy model (with revenue as the dependent variable). We see a similar pattern as for efficiency. With national synergies, revenue is significantly higher in the CCA formats. But with linear synergies it is significantly higher in SMRA. A summary of the results for efficiency and revenue across all auction formats and value models can be found in Table 5 and Table 6.

5. Conclusions

Much has been written about the design of efficient spectrum auctions in the past two decades (see e.g. Bichler and Goeree (2017) for an up-to-date overview). Traditionally, game-theoretic analyses and laboratory experiments have been used to analyze different auction formats. These methods have their limitations. In particular, spectrum auctions in the field typically have many licenses, complex activity rules, and spectrum caps. Such design elements are important, but typically ignored in theoretical and lab studies. Simulation studies complement theory and experiments. They allow economists to study alternative auction formats under exactly the same rules as used in the field. One contribution of the paper is the implementation of the SMRA and various versions of the CCA formats in a unified simulation framework, as well as an instance generator that estimates bidder valuations based on drop out bids from the Canadian 700MHz spectrum auction.

The economic environment in this simulation mirrors the Canadian market with all its institutional details, and it allows us to study the efficiency of wide-spread spectrum auction formats with different levels of synergies in the valuations of bidders. We assume that bidders maximize payoff in each round. This can serve as a reasonable approximation of bidder behavior in larger markets such as the Canadian auction. In any case, it is important to understand the average approximation ratio of simple auction algorithms in realistic environments when bidders bid straightforward.

The main results of the experiments go against wide-spread wisdom. Even high synergies do not always lead to higher efficiency in the combinatorial clock auctions compared to SMRA, and the relative efficiency ranking depends on the type of synergies. We analyzed two types of synergies motivated from observations in the field. In the "extreme national" synergy model synergies only occur when a bidder wins *all* licenses in a national package (and not if the bidder wins, say, 99% of the licenses). The extreme national synergies create the largest possible risk for a bidder who wants to aggregate licenses in the SMRA and, not surprisingly, the SMRA results in low efficiencies in this model. More moderate synergies occur when the marginal value of a license rises linearly with the number of licenses won. Surprisingly, under this assumption of "linear" synergies, the SMRA outperforms various versions of the CCA, in terms of efficiency as well as revenue. Overall, it is interesting to observe that the average efficiency loss in both models is remarkably low considering the simplicity of the algorithms and the worst-case approximation ratio of the allocation problem in combinatorial auctions.

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A. The Bidders' Value Models

We estimate value models for the individual bidders from the bid data in an attempt to get realistic valuations. While we do not aim to estimate the true valuations of bidders from the data, we want to get valuations which resemble those in the field. In an initial step, we preprocessed the data and based our estimates on the supplementary bids. We also removed a few outliers, bids that were substantially higher or lower than the other bids of a bidder, and which might have had strategic reasons.

Instead of a standard L2 linear regression, we used an L1 linear regression, which is constrained to have no intercept. L1 regressions are more robust against outliers (Andersen, 2008). In this regression, the package bid is the dependent variable, the vector of licenses included in a package bid describes exogenous variables. Such regressions were run for each bidder. The R^2 of the resulting models for the three national bidders Rogers, Bell, and Telus are 0.60, 0.89, and 0.96, respectively. We provide the value models in our data companion. These estimated valuations provide the means of a distribution describing the value of each type of license. For each valuation set used in a simulation, we draw different valuations for the individual licenses of different bidders. Synergies for packages were then modeled on top as described in our experimental design.

B. The Efficient Allocation

In what follows, we provide a quick introduction into the mixed integer program (MIP) that is used to compute the efficient allocation in all our experiments. A similar formulation (albeit only for a single bidder) can be used to compute a bidder's best response and is used in the simulation framework to compute the bidders' payoff-maximizing package in their straightforward bidding strategy.

The allocation of licences can be modeled as a multi-knapsack problem, in which each licence $i \in I$ with capacity or quantity $q(i)$ will be assigned to a bidder $j \in J$. Part of the objective function is the sum of the bidder's additive valuations $v_j^A(i)$ for a licence i times whether he actually won the licence $x_{i,j}$ (3) or not. Assignining at most $q(i)$ licences for each licence type is checked at (7). Each licence also has $e(i)$ eligibility points associated with it and which will be tested against bidder's maximum allowed eps $e_j^{\max}$ (10). Equations (8) and (9) apply the capping rules defined by the Canadian regulator. Note that the national bidders $J^{\mathrm{N}} \subset J$ are slightly more constrained across the available regions R. Each bidder can potentially have one or multiple non-additive demand vectors $r_{j,k}^N \in (\{-1\} \cup \mathbb{N}_0)^{|I|}$, where -1,0 and $\mathbb{N}$ signals indifference, should not receive, and the minimum quantity, respectively. If a demand vector is fulfilled, the equations (13) through (16) make sure the associated non-additive valuation $v_j^N(k)$ is added to the objective (4). This part of the algorithm is used to model the National Synergy Value Model. Similarly, a bidder can have an incremental bonus that reaches its maximum relative complementarity b_j^T, when all of the bidders relevant bonus items I_j^B reach the required quantity $r_j^B(i)$ (equation (6)). In order to stay in linear space, the relative factors were linearized, as seen in (6) and (17) to (19). Please refer to Table 4 for a summary of all involved parameters and variables.

Parameters	
I	Items
J	Bidders
J^{N}	national bidders
b_j^T	added complementarity (1.0 = 100 % = additivity)
e_j^T	total eps of the bidder's bonus package
$e(i)$	eps of item
$e_j^{\max}$	maximum eps of the bidder
$r_j^B(i)$	quantity j requires for bonus of i
$\mathrm{band}(i)$	band of item
R	available regions
$v_j^A(i)$	additive surplus
$v_j^N(k)$	nonadditive surplus
$r_{j,k,i}^N$	required quantity of i in bidder j's package k
$q(i)$	available quantity of i
Variables	
$x_{i,j} \in \mathbb{N}$	quantity bidder j receives of item i
$y_{j,k} \in \mathbb{B}$	the bidder won package k
$y_{j,k,i} \in \mathbb{B}$	variable for non-additive packages, item-wise
$b_{j,i_1,i_2} \in \mathbb{B}$	bonus active for i_1, i_2

Table 4: Sealed Model Parameters and Variables

$$
\begin{aligned}
\max \quad & \sum_{j \in J} \left(z_j^{add} + z_j^{na} \right) && && (2)\\
\text{s.t.} \quad & \sum_{i \in I} v_j^A(i) \cdot x_{i,j} = && z_j^{add} \quad \forall j \in J && (3)\\
& \sum_{k \in K_j} v_j^N(k) \cdot y_{j,k} + && && (4)\\
& \frac{b_j^T - 1}{\mathrm{ep}_j^T} \sum_{i_1 \in I_j^B} \sum_{i_2 \in I_j^B} r_j(i_1) && && (5)\\
& \cdot e(i_1) \cdot r_j(i_2) \cdot v_j^A(i_2) \cdot b_{j,i_1,i_2} \geq && z_j^{na} \quad \forall j \in J && (6)\\
& \sum_{j \in J} x_{i,j} \leq && q(i) \quad \forall i \in I && (7)\\
& \sum_{i \in I_r : \mathrm{band}(i) != \text{'DE'}} x_{i,j} \leq && 2 \quad \forall r \in R, \forall j \in J && (8)\\
& \sum_{i \in I_r : \mathrm{band}(i) \in \{\text{'BC'}, \text{'C1C2'}\}} x_{i,j} \leq && 1 \quad \forall r \in R, \forall j \in J^{\mathrm{N}} && (9)\\
& \sum_{i \in I} e(i) \cdot x_{i,j} \leq && e_j^{\max} \quad \forall j && (10)\\
& \sum_{i} y_{j,k,i} \leq && |I| \cdot (1 - y_{j,k}) \quad \forall j, \forall k && (11)\\
& 1 - \sum_{i} y_{j,k,i} \leq && |I| \cdot y_{j,k} \quad \forall j, \forall k && (12)\\
& r_{j,k,i}^N - x_{i,j} \leq && q(i) \cdot y_{j,k,i} \quad \forall j \in J, \forall k \in K_j, \forall i \in I : r_{j,k,i}^N > 0 && (13)\\
& 1 - r_{j,k,i}^N + x_{i,j} \leq && q(i) \cdot (1 - y_{j,k,i}) \quad \forall j \in J, \forall k \in K_j, \forall i \in I : r_{j,k,i}^N > 0 && (14)\\
& x_{i,j} \leq && q(i) \cdot y_{j,k,i} \quad \forall j \in J, \forall k \in K_j, \forall i \in I : r_{j,k,i}^N = 0 && (15)\\
& 1 - x_{i,j} \leq && q(i) \cdot (1 - y_{j,k,i}) \quad \forall j \in J, \forall k \in K_j, \forall i \in I : r_{j,k,i}^N = 0 && (16)\\
& r_j^B(i) \cdot b_{j,i,i} \leq && x_{i,j} \quad \forall j \in J, \forall i \in I && (17)\\
& b_{j,i_1,i_2} \leq && b_{j,i_1,i_1} \quad \forall j \in J, \forall i_1 \in I_j^B, \forall i_2 \in I_j^B : i_1 \neq i_2 && (18)\\
& b_{j,i_1,i_2} \leq && b_{j,i_2,i_2} \quad \forall j \in J, \forall i_1 \in I_j^B, \forall i_2 \in I_j^B : i_1 \neq i_2 && (19)
\end{aligned}
$$

$$
\begin{aligned}
x_{i,j} \in &\quad \mathbb{N}_0 && \forall j \in J, \forall i \in I \\
y_{j,k} \in &\quad \mathbb{N}_0 && \forall j \in J, \forall k \in K \\
y_{j,k,i} \in &\quad \mathbb{N}_0 && \forall j \in J, \forall k \in K, \forall i \in I
\end{aligned}
$$

C. Modeling the Exposure Risk

A limit to how much a bidder should expose himself can be directly included in the bidders individual bidding selection mixed integer problem. We do this by first deciding on an exposure factor exposure_j he should not surpass, i.e.:

$$\text{exposure}_j \geq \frac{z_j^{add} + z_j^{na}}{z_j^{add}}$$

We take the individual parts of the bidders target function z_j of the bidder, i.e., its additive part, z_j^{add} and the non-additive part z_j^{na}, which we want to limit. After reformulating the formula above we get the following inequality

$$z_j^{na} \leq (\text{exposure}_j - 1) \left(\sum_{i \in I} v_j^A(i) \cdot x_{i,j} \right)$$

which can then be integrated into the selection MIP to further limit the value of z_j^{na}.

D. Results Tables

What follows are the mean and standard deviation for all aggregate metrics.

	Additive	National-2.0	National-2.5	Linear-2.0	Linear-2.5
SMRA_{No}	0.9994 (0.0003)	0.7735 (0.0060)	0.6902 (0.0484)	0.9952 (0.0037)	0.9830 (0.0251)
$\mathrm{SMRA}_{+50\%}$	0.9994 (0.0003)	0.8130 (0.0040)	0.7194 (0.0319)	0.9954 (0.0033)	0.9952 (0.0035)
$\mathrm{SMRA}_{+100\%}$	0.9994 (0.0003)	0.8130 (0.0040)	0.7296 (0.0210)	0.9998 (0.0004)	0.9952 (0.0035)
SMRA_{Full}	0.9994 (0.0003)	0.8130 (0.0040)	0.7286 (0.0214)	0.9998 (0.0005)	0.9997 (0.0007)
SCCA(1)	0.9737 (0.0258)	0.8502 (0.0076)	0.8647 (0.0030)	0.8733 (0.0629)	0.8679 (0.0530)
SCCA(2)	0.9754 (0.0260)	0.8554 (0.0098)	0.8702 (0.0031)	0.8763 (0.0641)	0.8703 (0.0509)
CCA (clock)	0.9674 (0.0281)	0.8296 (0.0284)	0.8441 (0.0343)	0.8596 (0.0622)	0.8338 (0.0576)
CCA(0)	0.9742 (0.0192)	0.8445 (0.0225)	0.8544 (0.0272)	0.9642 (0.0132)	0.9648 (0.0148)
CCA(200)	0.9742 (0.0192)	0.8446 (0.0226)	0.8544 (0.0272)	0.9644 (0.0132)	0.9649 (0.0148)
CCA+(200)	0.9790 (0.0215)	0.9554 (0.0195)	0.9590 (0.0268)	0.9641 (0.0067)	0.9654 (0.0042)

Table 5: Efficiency - mean, (std. deviation)

	Additive	National-2.0	National-2.5	Linear-2.0	Linear-2.5
VCG	0.3820 (0.0038)	0.6182 (0.0078)	0.5634 (0.0071)	0.4879 (0.0043)	0.4946 (0.0043)
SMRA_{No}	0.4048 (0.0049)	0.4272 (0.0052)	0.3559 (0.0040)	0.4821 (0.0090)	0.4347 (0.0094)
$\text{SMRA}_{+50\%}$	0.4048 (0.0049)	0.4822 (0.0052)	0.4019 (0.0119)	0.4840 (0.0057)	0.4390 (0.0056)
$\text{SMRA}_{+100\%}$	0.4048 (0.0049)	0.4822 (0.0051)	0.4874 (0.0183)	0.5235 (0.0066)	0.4482 (0.0073)
SMRA_{Full}	0.4048 (0.0049)	0.4823 (0.0053)	0.4869 (0.0165)	0.5235 (0.0071)	0.5258 (0.0075)
SCCA(1)	0.3974 (0.0263)	0.5408 (0.0315)	0.4954 (0.0101)	0.4379 (0.0492)	0.4713 (0.0348)
SCCA(2)	0.3998 (0.0266)	0.5374 (0.0109)	0.5019 (0.0104)	0.4438 (0.0497)	0.4717 (0.0380)
CCA (clock)	0.3912 (0.0257)	0.5060 (0.0274)	0.4748 (0.0342)	0.3915 (0.0631)	0.3662 (0.0590)
CCA(0)	0.3098 (0.0256)	0.5769 (0.0211)	0.5055 (0.0084)	0.3665 (0.0269)	0.3932 (0.0229)
CCA(200)	0.3098 (0.0256)	0.5772 (0.0212)	0.5056 (0.0084)	0.3741 (0.0266)	0.3863 (0.0218)
CCA+(200)	0.3097 (0.0271)	0.5713 (0.0136)	0.5127 (0.0174)	0.3739 (0.0278)	0.3860 (0.0217)

Table 6: Revenue - mean, (std. deviation)