Spectrum Auction Design: Simple Auctions For Complex Sales

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Abstract

Following the successful PCS Auction conducted by the US Federal Communications Commission in 1994, auctions have replaced traditional ways of allocating valuable radio spectrum. Spectrum auctions have raised hundreds of billion dollars worldwide and have become a role model for market-based approaches in the public and private sectors. The PCS spectrum was sold via a simultaneous multi-round auction, which forces bidders to compete for licenses individually even though they typically value certain combinations. This exposes bidders to risk when they bid aggressively for a desired combination but end up winning an inferior subset. Foreseeing this possibility, bidders may act cautiously with adverse effects for revenue and efficiency. Combinatorial auctions allow for bids on combinations of licenses and thus hold the promise of improved performance. Recently, a number of countries worldwide have switched to the combinatorial clock auction to sell spectrum. This two-stage auction uses a core-selecting payment rule. The number of possible packages a bidder can submit grows exponentially with the number of licenses, which adds complexity to the auction. For larger auctions with dozens of licenses bidders cannot be expected to reveal all their valuations during such an auction. We analyze the impact of two main

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design choices on efficiency and revenue: simple "compact" bid languages versus complex "fully expressive" bid languages and simple "pay-as-bid" payment rules versus complex "core-selecting" payment rules. We consider these design choices both for ascending and sealed-bid formats. We find that simplicity of the bid language has a substantial positive impact on the auction's efficiency and simplicity of the payment rule has as a substantial positive impact on the auction's revenue. The currently popular combinatorial clock auction, which uses a complex bid language and payment rule, achieves the lowest efficiency and revenue among all treatment combinations.

Keywords: spectrum auctions, bid languages, laboratory experiments

1. Introduction

The 1994 sale of radio spectrum for "personal communication services" (PCS) marked a sharp change in policy by the US Federal Communications Commission (FCC). Before turning to auctions the FCC had allocated valuable spectrum on the basis of comparative hearings (also known as "beauty contests") and lotteries. Nobel laureate Ronald Coase long advocated that market-based mechanisms would improve the allocation of scarce spectrum resources, but his early insights were ignored for decades [1]. The PCS auction raised over six hundred million dollars for the US treasury and it was widely considered a success. Several authors discuss the advantages and disadvantages of auctions and beauty contests for allocating scarce spectrum [2, 3, 4]. For example, some argue that financially strong bidders might have advantages over weaker bidders in an auction, while others argue that with efficient capital markets such differences should be less of a concern. Nowadays, spectrum is predominantly assigned by auction, both in the US and elsewhere [5, 6], and in this paper we focus on questions of auction design.

The simultaneous multi-round auction (SMRA), which was designed for the US FCC in the early 90's has been the standard auction format for selling spectrum world wide for many years. It auctions multiple licenses for sale in parallel and uses simple activity rules which forces bidders to be active from the start. Despite the simplicity of its rules there can be considerable strategic complexity in the SMRA when there are synergies between licenses that cover adjacent geographic regions or between licenses in different frequency bands. Bidders who compete aggressively for a certain combination of licenses risk being exposed when they end up winning an inferior subset at high prices. When bidders rationally anticipate this *exposure problem*, competition will be suppressed with adverse consequences for the auction's performance. The exposure problem has led auction designers to consider combinatorial auctions, which enable bidders to express their preferences for an entire set of licenses directly. In fact, the design of spectrum auctions is seen as a pivotal problem im multi-object auction design and successful solutions are a likely role-model for other public or private sector auctions such as transportation or industrial procurement.

Since 2008, the combinatorial clock auction (CCA) has been used by regulators in various countries such as the Austria, Australia, Canada, Denmark, Ireland, the Netherlands, and Switzerland to sell spectrum.[1] The CCA combines an ascending auction where individual license prices rise over time (clock phase) in response to excess demand, with a sealed-bid supplementary phase. In addition, the auction uses a complex activity rule to set incentives for bidders to bid actively from the start [10]. Unlike the SMRA, bidders can demand combinations of licenses as well as individual licenses.

Combinatorial auctions can employ different types of bid languages, such as OR and XOR languages. Both allow bidders to submit indivisible bids on packages. For example, if a bidder bids on packages $\{A, B\}$ and $\{C, D\}$, he would only be assigned one of the packages at most with an XOR language. With an OR language he might win both packages. This way, the number

[1] A single stage combinatorial clock auction has been proposed in [7]. Such a single-stage ascending clock auction format was used in Nigeria [8], for example. In contrast, we discuss the two-stage combinatorial clock auction that has been used in spectrum auctions throughout the world in the past five years [9].

of different bids is reduced substantially. However, if a bidder only wants to win one of the two packages and not both, he cannot express this in a pure OR language. Actually, the OR language can only express superadditive valuations.

The CCA has employed an XOR bid language so far, but this comes at the price of high communication complexity.[2] With thirty licenses the number of possible combinations already exceeds a billion, which are far too many for bidders to express their values for.[3] This can lead to inefficiencies because the winner-determination algorithm allocates the spectrum as if missing bids for certain combinations reflect zero values for the bidders. Often the number of possible bids per bidder even has to be capped to a few hundred in order to keep the winner-determination problem feasible. In the bid data that was recently released by Ofcom for the CCA that was conducted in the UK in 2013 bidders submitted bids on between 8 and 62 packages in the supplementary round from 750 possible package bids considering the spectrum caps.[4] It is unlikely that bidders had a zero value for all the other packages.

In spectrum auctions it is typically common knowledge what combinations of licenses generate the most synergies.[5] In this paper, we study how the in-

[2] This is separate from the issue of computational complexity for the auction designer, i.e., how to determine which bids are winning, which is known to be an NP-hard computational problem. Nisan [11] point out that for fully efficient allocations and general valuations the communication requirements grow exponentially.

[3] Spectrum auctions with dozens of licenses have been conducted in Austria, Australia, Switzerland, the Netherlands, Ireland, and the UK. For example, in the 2012 auction in the Netherlands, 41 spectrum licenses in the 800 MHz, 900 MHz and 1800 MHz bands were sold. Switzerland auctioned 61 licenses distributed over 11 bands in 2012. Canada used a CCA for 98 licenses in 2014. Although not all packages will have a value for bidders in such auctions, large national bidders will not be able to submit bids for all packages with positive value in auctions with this many items.

[4] http://stakeholders.ofcom.org.uk/spectrum/spectrum-awards/awards-archive/completed-awards/800mhz-2.6ghz/auction-data/

[5] For example, there is high complementarity within the 800 MHz band in most European auctions, where a package of two licenses often has much higher value than two times the value of a single license. For the new LTE mobile communication standard, telecom companies typically aim for four adjacent blocks of spectrum (i.e., 20 Mhz) in higher bands to fully

troduction of a simple bid language, tailored to capture the main synergies, affects the performance of multi-band spectrum auctions. Our bid language allows bidders to specify either-or bids on packages within a band (XOR) while bids for packages in different bands are considered additive (OR). This way, the number of possible bids is reduced substantially. Although elements of the bid language can be used in practice, we do not suggest there is a one-size-fits-all bid language. Rather, we want to understand the potential benefits of such an OR-of-XOR bid language over a fully expressive one. Interestingly, the design of compact bid languages has not been an issue in the design of spectrum auctions in different countries and a fully expressive XOR bid language has always been used for the CCA.

Besides the bid language, another defining feature of the CCA is the *core-selecting payment rule*. Theoretical considerations for this payment rule are based on the Vickrey-Clarke-Groves (VCG) mechanism, which has a simple dominant strategy for bidders to submit their valuations truthfully. The VCG mechanism, however, can lead to outcomes where the winners pay less then what losing bidders are willing to pay with their bids.

A simple example should illustrate the problem. Suppose three bidders 1, 2 and 3 bid on two items A and B. Bidder 1 is only interested in item A and the bundle $\{A, B\}$ for \$2 since we assume free disposal. Bidder 2 is only interested in B and the bundle of both items for \$2, and bidder 3 is willing to pay \$0 for each item and \$2 for the bundle. Bidders 1 and 2 are winners and in a VCG auction they get a discount equaling their marginal contribution to the overall revenue. Bidder 1 would win A and pay his bid price of \$2 minus (\$4-\$2), i.e., the difference in revenue with and without him. This means bidder 1 would pay zero and likewise also the second bidder would pay zero in a VCG auction resulting in zero revenue, although there was another losing bidder, who expressed a willingness to pay \$2.

To avoid such "non-core" outcomes with respect to the bids, the core-

leverage the new standard.

selecting payment rule has been used in the CCA. This payment rule is sufficiently complex that it generally does not allow for a game-theoretic analysis and its outcomes can appear non-transparent as small changes in the package bids selected by the bidders can lead to substantial variations in the payments.[6] Moreover, the payments are not known until after the auction, which precludes bidders from reporting to management about the progress of the auction and about expected payments. These issues do not arise with a simple pay-as-bid payment rule as used, for instance, in the Romanian spectrum auction in 2012.

Avoiding uncompetitively low revenue as is possible in a VCG mechanism was one of the original design goals of core-selecting payment rules [13]. Arguably, revenue is an important result in spectrum auctions even though it rarely is an official design goal. Note that efficiency cannot be analyzed in the field where the bidders' valuations stay private. Transparency of the auction process and the law of one price (for a license across all bidders) are additional design goals apart from efficiency and revenue that matter in spectrum auction design and there are trade-offs between these goals. Both, the CCA and the VCG mechanism do not satisfy the law of one price and for example in Switzerland one of the bidders had to pay almost 482 million Swiss Francs and another bidder close to 360 million Francs although they won a similar set of licenses.[7] These problems have led to discussions among regulators and telecoms on pros and cons of different auction designs used for selling spectrum. In particular, stakeholders need to understand the impact of different bid languages and different payment rules on the overall efficiency and revenue of the auction.

We have implemented the two-stage CCA with all the activity, allocation, and core-selecting payment rules as it is used in the field, but also the alternative treatments and analyzed them in lab experiments. The different treatments of our experiment allow us to measure how auction revenue varies when using the

[6]For a simplified setting, Goeree and Lien [12] show that the "core selecting" payment rule may result in prices that are further from the core than Vickrey prices.

[7]Results of the Swiss auction can be found at http://www.news.admin.ch/NSBSubscriber/message/attachments/26004.pdf

pay-as-bid or core-selecting payment rule. We consider the treatment variations, simple versus complex bid language and simple versus complex payment rule, for both ascending and sealed-bid formats. We find that simplicity of the bid language has a substantial positive impact on the auction's efficiency and simplicity of the payment rule has as a substantial positive impact on the auction's revenue.

2. Experimental design

In what follows, we characterize treatment variables, in particular the value model, the bidding language, and the auction formats, before we discuss details of the organization of our experiments.

2.1. The value model

In this paper we will draw on the multi-band value model used in earlier experiments [10], which has four bands with 6 licenses each. Within a band, each individual block has the same value for bidders so that there are essentially $7^4 - 1 = 2,400$ different packages. The structure of the value model and the distribution of the block valuations of all bands are known to all bidders. In particular, band A is of high value to all bidders and bands B, C, and D are less valuable. Bidders receive base valuations for items in each band. Base valuations are uniformly distributed: v_A was in the range of $[100, 300]$ while v_B, v_C, and v_D were in the range of $[50, 200]$. Furthermore, bidders have complementary valuations for bundles of blocks within bands, but not across bands. In all bands, bundles of two blocks resulted in a bonus of 60% on top of the base valuations, while bundles of three or more blocks resulted in a bonus of 50% for the first three blocks. For example, if the base value was 100 experimental Francs, then the valuation for two blocks was 320, for three blocks 450, and for four blocks 550. Although the value models resemble characteristics of actual spectrum sales, this was not communicated to the subjects in the lab to maintain a neutral framing.

2.2. Bid languages

The bidding language in a CA specifies the kinds of bids that can be placed by a bidder. Under the fully expressive XOR bid language, bids can be placed on any of the 2,400 different packages with the understanding that at most one of the bids can become winning. This bid language has been used in all spectrum auctions so far, as it allows expressing all possible preferences including complements and substitutes. As already introduced, this expressivity comes at the price of an exponential number of possible packages. There have been attempts to design bid languages allowing bidders to express their preferences with a lower number of parameters. For example, the OR* language extends the OR language to allow a bidder to introduce some number of bidder-specific dummy items. The dummy items have no value, but they allow the bidder to constrain the sets of bids that can be selected by the auctioneer. This can substantially reduce the number of bids that a bidder has to submit [14]. Apart from such generic bid languages, domain-secific bid languages have been proposed, which leverage common knowledge about the utility functions of bidders in specific markets [15].

In our experiments, we use an OR-of-XOR bid language, which draws on the observation that typically there are high synergies among licenses within a band, but lower synergies across bands. In the experiments bids could be submitted on 2, 4, and 6 lots only in each of the bands and at most one of the bids within a band could become winning. However, a bidder could win multiple bids in different bands, i.e. we use an OR bid language across bands. Overall, bidders can submit $3 * 4 = 12$ bids in each round, and win a maximum of 4 bids (one bid per band) in this OR-of-XOR language (see Figure 1). In our value model there were no cross-band synergies. Even if there were synergies across bands, bidders can often handle the remaining exposure risk well. Overall, the bid language and the value model might differ for specific applications, but the experiments allow us to estimate the differences in efficiency of a compact bid language compared to an XOR bid language, which has been used in spectrum auctions so far.

2.3. Treatment structure

We analyze two variations, simple (S) and complex (C), of the bid language and payment rule. In particular, we consider a compact bid language versus a fully expressive bid language, and a pay-as-bid versus a bidder-optimal core-selecting payment rule. We do so for both ascending (A) and sealed-bid (SB) auctions. The different treatments are denoted F_{LP} where $F = \{A, SB\}$ denotes the format and the subscripts $L = \{S, C\}$ and $P = \{S, C\}$ indicate the bid language and payment rule respectively (see Table 2). For example, the CCA is denoted A_{CC} while SB_{SS} denotes a sealed-bid auction with a compact bid language and a pay-as-bid payment rule. The only ascending auction format with a fully expressive bid language we consider is the A_{CC} (and not A_{CS}) since it is the incumbent standard, this means the CCA.[8] Instead of the A_{CS} we include the SMRA, which used to be the standard and also has a simple pay-as-bid payment rule and a (super) compact bid language, i.e. OR bidding within and across bands.

The sealed-bid formats are straightforward in that bids can be submitted only once, after which the winner-determination problem is solved and prices are computed. In contrast, the ascending auctions consist of an unknown number of rounds and at the start of each round ask prices for all licenses are announced. Based on these ask prices, bidders report whether they are interested in 0, 2, 4, or 6 licenses in each of the four bands. If there is excess demand (i.e., if the combined demand of all bidders exceeds the number of licenses available) in at least one band, a new round starts with higher ask prices for the bands with excess demand. Prices in the first round are set to 100 for items in the A band and to 50 in the B, C, and D bands. The price increment in the A band is 20 while in the B, C and D bands it is set to 15. A bidder has to submit at least one bid in each round to bid again for bundles in the next round. When there is no

[8] Ascending auction formats with an XOR bid language, a pay-as-bid payment rule and non-linear and personalized ask prices have already been tested in the lab [16], but the number of auction rounds renders them impractical for larger auctions with more than 10 items.

more excess demand in any of the bands the winner determination problem is solved considering *all* bids submitted during the entire auction. If the computed allocation does not displace an active bidder from the last round the auction terminates, otherwise the price is incremented in those bands where a bidder was displaced to give now losing bidders a chance to improve their bid.[9]

We put great emphasis on the external validity of the experiments. The value models were modeled after real-world auctions, and bidders were provided decision support in selecting their packages based on value or payoff after each round which reflects practices in the field. Still, there might be phenomena in the field that we do not observe in the lab. For example, in our experiments we provided strong incentives to maximize payoff. Bidders in the field might be spiteful and try to block other competitors from getting their preferred allocation or drive up their prices on items other bidders desire. In a combinatorial clock auction they might also try to increase payments of others by submitting bids with a high probability of losing [10]. Such issues are unlikely in the lab. Still, the lab results help understand many aspects bidder behavior such as restricted bundle selection and its consequences on auction efficiency and revenue.

2.4. Procedures and organization

We used the same sets of value draws ("waves") across treatments to reduce performance differences due to the random draws. Each wave was used to run four different auctions, which combined define one session. We ran between subjects experiments with four bidders in each session. The experiments were conducted from June 2012 to March 2013 with subjects from computer science, mathematics, physics, and mechanical engineering. The subjects were recruited via e-mail. Each subject participated in a single session only.

The sessions with the ascending auction took around four hours and the sealed bid auctions between 1.5 and 2.5 hours. At the start of each session the

[9]This procedure is in line with the single stage combinatorial clock auction [7]. A theoretical analysis of this auction format can be found in [17].

environment, the auction rules and all other relevant information was explained to the participants. The instructions were read aloud and participants had to pass a test before they were admitted to start the experiment.

A spreadsheet tool was provided to subjects to analyze payoffs and valuations in each round. This tool showed a simple list of available bundles, which could be sorted by bundle size, bidder individual valuations, or payoffs based on current prices in the ascending auction formats. At the start of each auction, subjects received their individual value draws, information about the value distributions and their synergies for certain bundles. Each round in the ascending auction took 3 minutes. The time given to the subjects in the sealed bid formats varied between 20 and 25 minutes (although subjects could always ask for more time when needed).

After all four auctions were completed, subjects were paid. The total compensation consisted of a 10 Euro show up fee and an auction reward, which was calculated as a 3 Euro participation reward plus the auction payoff converted to Euros at a 12:1 ratio. Negative payoffs were deducted from the participation reward. To compensate for the different durations of the ascending and sealed-bid auction formats, and for the differences in earnings stemming from the payment rules, we paid two out of four randomly drawn auctions in A_{SC}, three out of four in A_{SS}, 1.5 out of four auctions in SB_{CS} and SB_{SS},[10] and one out of four auctions in SB_{CC} and SB_{SC}. On average, each subject earned 70.94 EUR in A_{SC} and 69.75 EUR in A_{SS}, 37.69 EUR in the sealed bid auction with compact bid language (SB_{SC}, SB_{SS}) and 42.16 EUR in the sealed bid expressive auction (SB_{CC}, SB_{CS}).

3. Results

We will first present aggregate results, i.e., efficiency and revenue of the different auction formats, and then discuss individual bidder behavior. For the

[10]The first auction that was drawn was paid fully and for the second auction only half the payoff.

pairwise comparisons of various metrics we use the rank sum test for clustered data to reflect that the auctions were conducted in sessions with the same set of subjects [18].

3.1. Efficiency and revenue

We compare auction formats in terms of *allocative efficiency*

$$E = \frac{\text{actual surplus}}{\text{optimal surplus}} \times 100\%$$

and in terms of *revenue distribution*

$$R = \frac{\text{auctioneer's revenue}}{\text{optimal surplus}} \times 100\%$$

which shows how the resulting total surplus is distributed between the auctioneer and the bidders. Optimal surplus describes the resulting revenue of the winner-determination problem if all valuations of all bidders were available, while actual surplus considers the true valuations for those packages of bidders selected by the auction. In contrast, auctioneer's revenue used in the revenue distribution describes the sum of the bids selected by the auction, not their underlying valuations.

Regulators typically aim for competition and high efficiency in the downstream wireless telecommunications market after the auction. Note that this is different from allocative efficiency of the auction outcome, which has been a primary concern in auction theory. Proponents of using auctions as a means to allocate spectrum argue that allocative efficiency of an auction places spectrum with those who value it most and who are therefore likely to develop it most effectively. Of course, the result of an efficient auction could be a monopoly, which is why auction designs sometimes include set aside blocks for new entrants and license acquisition limits (aka. caps) for bidders [2, 19]. In this paper, we assume sufficient competition in a market of telecom operators and analyze allocative efficiency as a desirable goal.

Although, high revenue is rarely an official design goal for regulators, it is always an issue after spectrum auctions in the absence of bidders' true valuations. Whether high revenue is seen as another goal in addition to efficiency or low payments for the bidders are seen as desirable depends on the overall telecommunications policy [2, 4]. In any case, it is important to understand the potential impact of payment rules on the revenue of an auction.

Result 1. *(i) Formats with a compact bid language are more efficient than those with a fully expressive language. To some extent the efficiency loss with a fully expressive bid language is due to the fact that items remain unsold, which does not happen with a compact bid language. (ii) Among the formats with a fully expressive bid language there are no efficiency differences. (iii) Among the formats with a compact bid language only the SMRA yields significantly, albeit not substantially, higher efficiency.*[11]

Result 1 is illustrated in Figure 2 and Table 2. The intuition behind the efficiency loss with fully expressive bid languages is that few bids among the 2,400 possible bids are selected (see Sections 2.2 and 2.3). The winner-determination algorithm assigns zero value to all packages not bid for, which distorts from the optimal allocation especially when the submitted bids create a fitting problem. Somewhat surprisingly, the SMRA comes out ahead despite the substantial complementarities within bands. Bidders did a good job in dealing with the resulting exposure risk, with high-value bidders taking more exposure risk and low-value bidders less.

A multiple linear regression confirms the impact of bid language (compact or fully expressive) on efficiency, while the payment rule (core-selecting or pay-as-bid) and the format (ascending or sealed-bid) have no significant effect (see Table 3).

Result 2. *Formats with a pay-as-bid payment rule yield higher revenue than those with a core-selecting payment rule. Among the formats with a pay-as-bid payment rule only the SMRA yields significantly and substantially less revenue.*

[11] In more detail, SMRA $\succ^* A_{SC} \sim SB_{SC} \sim A_{SS} \sim SB_{SS} \succ^* SB_{CC} \sim A_{CC} \sim SB_{CS}$, where $\sim$ indicates an insignificant order, $\succ$ indicates significance at the 10% level, $\succ^*$ indicates significance at the 5% level, and $\succ^{**}$ indicates significance at the 1% level.

Among the formats with a core-selecting payment rule those with a fully expressive bid language yield significantly and substantially less revenue.[12]

Support for result 2 can be found in Figure 2 and Table 2. The higher revenue for pay-as-bid sealed-bid auction formats might be explained by risk aversion. Auction format, bid language, and payment rule all have a significant impact on auctioneer revenue, see Table 4.

3.2. Bidder behavior in ascending auctions

Result 3. *Bidders in an ascending auction with a compact bid language select their bundles mainly based on payoff. Bidders did not only bid on their highest valued bundles, but on 72.9% of all bundles with a positive payoff. The payment rule did not have an impact on bundle selection. A fraction of 7.83% of all bids were above value in the* A_{SC} *auction compared to only 0.32% in the* A_{SS} *auction. In the supplementary phase of the two-stage CCA (*A_{CC}*) only a small fraction (0.06%) of the 2,400 possible bids were submitted.*

Note that in the clock phase of the CCA bidders are only allowed to submit a single package bid per round. Figure 3 shows how many bids were submitted on the bundle with the highest payoff (dark grey), the second and third highest payoff, and on how many bundles with a positive payoff were not bid on (light grey). The three bars summarize the distribution of such bids in the first, middle, and final third of all auction rounds (recall that the number of rounds varies across auctions). The two panels highlight that bidders did not only bid on the payoff maximizing bundle. Initially, they even submit more bids on bundles with the second or third highest payoff. We conjecture that bidders compare valuations rather than payoffs in the initial rounds.

Bids were frequently above value with the core-selecting payment rule, which might be due to the fact that the payment is lower than the submitted bid in

[12]In more detail, $SB_{SS} \sim SB_{CS} \sim A_{SS} \succ^* \text{SMRA} \succ A_{SC} \sim SB_{SC} \succ^* SB_{CC} \succ^* A_{CC}$.

this case.

3.3. Bidder behavior in sealed-bid auctions

Result 4. *Bidders in core-selecting sealed-bid auctions with a compact bid language bid on all possible bundles. Bidders in sealed-bid auctions with a fully expressive bid language bid only on 2.42% of all 2,400 possible packages. There was more bid shading with the pay-as-bid payment rule compared to the core-selecting payment rule.*

Figure 4 and Table 5 provide support for this result. We also estimated a linear regression with valuation as a covariate to explain bid prices (and bidder ID to control for unobserved heterogeneity among bidders). The intercept (α) and the slope (β) of the bidding function can be found in Table 6. The β coefficients are lower for pay-as-bid auctions, which indicates higher bid shading for higher valuations. The estimation results are shown by the dashed lines in Figure 4.

4. Discussion and Conclusions

The CCA is being increasingly used by regulators world-wide to sell spectrum licenses in multi-band auctions where bidders can submit bids on thousands or millions of different packages. The large number of possible bids introduces communication complexity into the auction, and it seems realistic to assume that bidders will typically submit bids only for a much smaller subset. Since the winner-determination algorithm assumes all other packages have zero value, the missing bids problem can have adverse effects for the auction's efficiency and revenue. This missing bid problem arises in any combinatorial auction that uses a fully expressive (XOR) bid language unlike, for instance, in the SMRA that employs a simpler OR language. Regulators therefore face a trade-off between the SMRA's exposure problem and the CCA's communication complexity, both of which negatively impact the auction's efficacy.

In this paper we consider a middle-ground solution that aims to mitigate both the exposure problem and communication complexity. In particular, we analyze a bid language that drastically reduces the number of possible bids that can be submitted. First, the bid language assumes bids in different bands are additive, like in the SMRA, so that across bands multiple bids can be winning. In addition, we allow only for bids on packages of 2, 4, or 6 licenses within a band and at most one such bid can be winning. This reduces the number of possible bids from 2,400 to 12. Although, bid languages will be different depending on the application and there might be some complementarities across bands as well, the experimental results demonstrate that a simpler compact bid language yields significantly and substantially higher efficiency levels compared to a complex and fully expressive XOR bid language.

The results of our experiments do not suggest that SMRA always outperforms combinatorial auctions in markets with many licenses. Complementarities might be such that the exposure problem for large bidders creates a substantial strategic problem. Actually, in spectrum auctions with many regional licenses such as in Canada or the USA nationwide carriers will have preferences for the availability of package bids. The results of the experiments do suggest, however, that a fully expressive XOR bid language leads to substantial efficiency losses for larger combinatorial auctions. Even though it will take extra effort to design an appropriate bid language and find an agreement among the stakeholders in an auction, this design decision is essential and it must not be ignored.

Besides complexity of the bid language we also studied how complexity of the payment rule affects auction performance. In particular, we compare a pay-as-bid rule to the core-selecting payment rule that underlies the CCA. We find that auction revenue is substantially higher with the simpler pay-as-bid rule. The pay-as-bid rule avoids uncertainty about how much a bidder has to pay for a bid at the end of an auction, if this bid becomes winning. Taken together our results underline the benefits of simplicity – both of the bid language and the payment rule.

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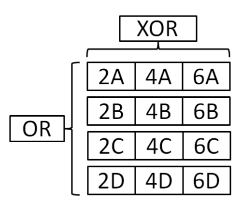

Figure 1: Compact bid language.

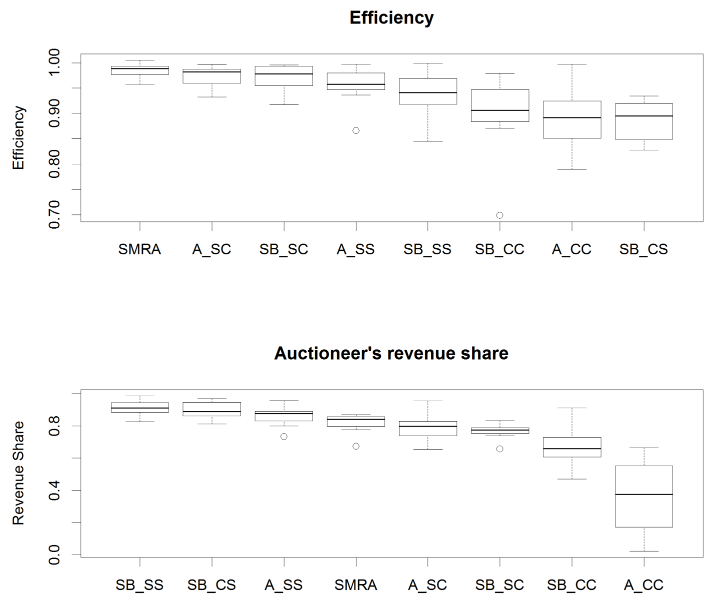

Figure 2: Efficiency and Revenue in the different auction formats.

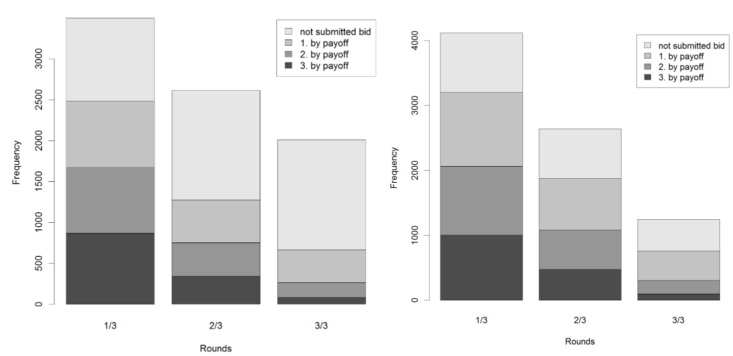

Figure 3: Distribution of bids by payoff in the A_{SS} (left) and A_{SC} (right) auction

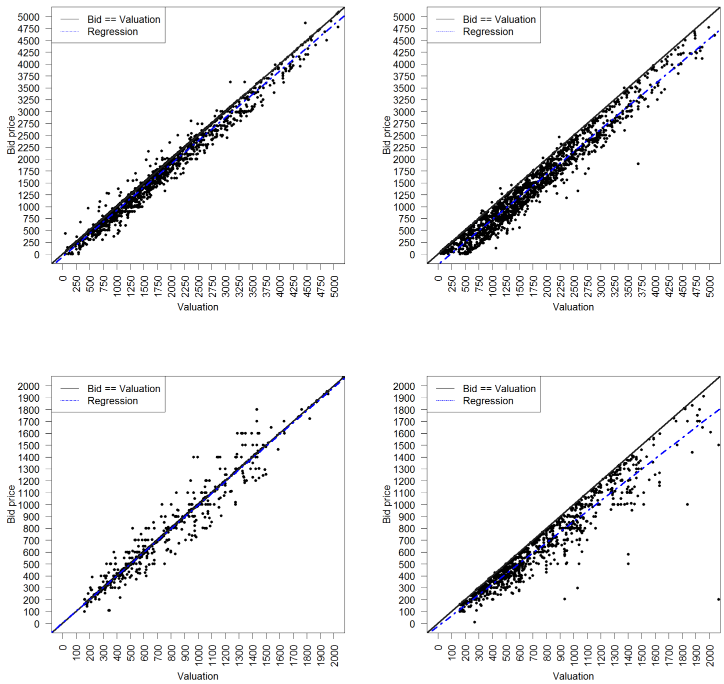

Figure 4: Bid shading in the auctions with core-selecting (left) and pay-as-bid auction (right) with a fully expressive bid language (top) and a compact language (bottom).

Treatment	Auction format	Bid language	Payment rule	Auctions
1 ($SMRA$)	ascending	single-item	simple	16
2 (A_{CC})	two-stage	complex	complex	16
3 (SB_{SC})	sealed-bid	simple	complex	16
4 (SB_{SS})	sealed-bid	simple	simple	16
5 (SB_{CS})	sealed-bid	complex	simple	16
6 (SB_{CC})	sealed-bid	complex	complex	16
7 (A_{SC})	ascending	simple	complex	16
8 (A_{SS})	ascending	simple	simple	16

Table 1: Treatment structure of the experiments

Auction	E	R	Unsold licenses
SMRA	98.51%	81.96%	0
A_{SS}	95.92%	86.62%	0
A_{SC}	97.26%	78.96%	0
A_{CC}	89.33%	37.41%	1.25 (5.2%)
SB_{SS}	94.33%	91.05%	0
SB_{SC}	97.21%	77.28%	0
SB_{CS}	88.56%	89.62%	0.82 (3.4%)
SB_{CC}	91.76%	65.53%	0.31 (1.3%)

Table 2: Aggregate measures of auction performance

Coefficients	Estimate	Pr(> $\|t\|$)
Intercept	0.9759	$< 2e-16$
XOR bid language	-0.0728	$1.36e-15$
Pay-as-bid payment rule	-0.0104	0.165
Auction format	-0.0081	0.279

Table 3: Impact of bid language, payment rule, and auction format on efficiency (adjusted $R^2 = 0.4239$).

Coefficients	Estimate	Pr(> $\|t\|$)
Intercept	0.6656	$< 2e-16$
XOR bid language	-0.1738	$3.93e-14$
Pay-as-bid payment rule	0.1794	$7.58e-13$
Auction type	0.1435	$2.89e-09$

Table 4: Impact of bid language, payment rule, and auction format on auctioneer's revenue (adjusted R^2=0.5827).

Format	truthful	overbidding	underbidding
SB_{SS}	0%	0.99%	99.01%
SB_{CS}	0%	1.23%	98.77%
SB_{SC}	32.34%	22.05%	45.61%
SB_{CC}	18.11%	4.55%	77.34%

Table 5: Truthful bidding in sealed-bid auctions

	α	β	p-value	adjusted R^2
SB_{SS}	0.5601	0.8834	0.0086	0.917
SB_{CS}	-0.0129	0.953	0.0033	0.986
SB_{SC}	-76.3868	0.9921	0.0056	0.975
SB_{CC}	-0.5637	0.9736	0.0029	0.986

Table 6: Estimated bid functions: $b = \alpha + \beta v$.