On the Robustness of Non-Linear Personalized Price Combinatorial Auctions

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Abstract

Though the VCG auction assumes a central place in the mechanism design literature, there are a number of reasons for favoring iterative combinatorial auctions (ICAs). Several promising ICA formats were developed based on primal-dual and subgradient algorithms. Prices are interpreted as a feasible dual solution and the provisional allocation is interpreted as a feasible primal solution. iBundle(3), dVSV and Ascending Proxy auction result in VCG payoffs when the coalitional value function satisfies buyer submodularity and bidders bid straightforward, which is an ex-post Nash equilibrium in this case. iBEA and CreditDebit auctions do not even require the buyer submodularity and achieve the same properties for general valuations. Often, however, one cannot assume straightforward bidding and it is not clear from the theory how these non-linear personalized price auctions (NLPPAs) perform in this case. Robustness of auctions with respect to different bidding behavior is a critical issue for any application. We conducted a large number of computational experiments to analyze the performance of NLPPAs with respect to different bidding strategies and valuation models. We compare NLPPAs with the VCG auction and with ICAs with linear prices, such as ALPS and the Combinatorial Clock auction. While NLPPAs performed very well in case of straightforward bidding, we observe problems with revenue, efficiency, and speed of convergence when bidders deviate.

Key words: combinatorial auction, primal-dual auction, subgradient auction, allocative efficiency, computational experiment, simulation

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1. Introduction

"Experience in both the field and laboratory suggest that in complex economic environments iterative auctions [...] produce better results than sealed bid auctions" (Porter et al. 2003). Several authors have tried to develop indirect auctions with strong incentive properties to overcome these problems, as in iterative combinatorial auctions, bidders don't have to reveal all their true preferences in one round as would be necessary in Vickrey-Clarke-Groves (VCG) mechanisms (Parkes and Ungar 2000, Ausubel and Milgrom 2006b). Iterative Combinatorial Auctions (ICAs) are also strategically simpler than "first-price sealed-bid" auction designs (Vickrey 1961).

The goal of achieving VCG payoffs in a CA is two-fold. The VCG mechanism is the unique auction that has a dominant-strategy property, leads to efficient outcomes, and takes zero payment from losing bidders (Green and Laffont 1977, Ausubel et al. 2006, p.93). Though the VCG auction assumes a central place in the mechanism design literature, its results are outside of the core, when bidders are not substitutes (see Definition 5). If this is the case, the seller's revenue can be uncompetitively low, and opens up bidder non-monotonicity problems, and possibilities for collusion and shill-bidding (see Ausubel and Milgrom (2006b), Day and Milgrom (2007) for a more detailed discussion). Bidder monotonicity means, that auctioneer revenue cannot decrease with additional bidders.

Two main approaches have been discussed in the literature for the design of ICAs. Some authors try to maintain linear prices at the expense of some levels in efficiency (Rassenti et al. 1982, Porter et al. 2003, Kwasnica et al. 2005, Bichler et al. 2009). For example, pseudo-dual prices describe an approach, where integer constraints of the winner determination problem are relaxed and linear prices are derived from a restricted dual problem. Another school of thought uses non-linear personalized ask prices. This means that there is an individual ask prices for each bundle and each bidder in the worst case. The approach is based on an extended linear program of the winner determination problem introduced by Bikhchandani and Ostroy (2002) that always implements an integral solution, even without integer restrictions on variables. Consequently, the non-linear and personalized ask prices derived from the dual variables will lead to competitive equilibrium, maximizing allocative efficiency when bidders follow a *straightforward* bidding strategy. Straightforward bidding describes a strategy in which, in each round, the bidder submits the minimum bid on the bundles maximizing his payoff at the current ask prices.

While the original formulation in Bikhchandani and Ostroy (2002) leads to an exponentially large number of additional variables, it has inspired a number of practical auction designs. Primal-dual algorithms and subgradient algorithms have been used as a conceptual framework to design iterative combinatorial auctions such as iBundle(3) (Parkes and Ungar 2000), the Ascending Proxy Auction (Ausubel and Milgrom 2006a), and dVSV (de Vries et al. 2007). All three auction designs result in VCG payments, when the *bidder submodularity condition* is satisfied and bidders follow a straightforward strategy. It can also be shown that there are incentives to do so: a straightforward bidding strategy is an ex post equilibrium as long as the submodular valuations condition is satisfied (Mishra and Parkes 2007). An ex-post equilibrium assumes, however, that all bidders bid straightforward, i.e., they play their best-response strategy. Given, that bidders do not know a priori, whether bidder submodularity conditions hold, this is a strong assumption.

Although, de Vries et al. (2007) show that for private valuation models without restrictions ascending combinatorial auctions cannot achieve VCG payoffs, newer approaches try to overcome this by extending the definition of ascending price auctions. For example, Ausubel (2006) uses multiple price paths. The CreditDebit auction by Mishra and Parkes (2007), based on the dVSV auction, calculates discounts on the quoted prices, and in a similar way, iBEA is based on iBundle(3). These formats will terminate with VCG payments for general valuations. In the following, we will call iBundle, the Ascending Proxy Auction, dVSV, CreditDebit, and iBEA *non-linear personalized price auction designs (NLPPAs).*

Clearly, NLPPAs can be considered a fundamental contribution to the combinatorial auction theory, as they describe iterative auction designs that are fully efficient. However, they are based on a number of assumptions. In particular, straightforward bidding might not hold in practical settings where bidders have bounded rationality, given that bidders don't know, whether the submodularity condition holds, and there is a huge number of bundles a bidder has to deal with. Recent experimental work has actually shown that bidders did not follow a pure best-response strategy, even in simple settings with only a few items (Scheffel et al. 2009). Therefore, it is important to understand their performance in case of non-straightforward bidding strategies, when bidders either cannot follow such a strategy for computational or cognitive reasons, or deliberately choose another strategy.

One of the beauties of double auction markets is their robustness against simple, even random bidding strategies, as shown by Gode and Sunder (1993). Similarly, we will introduce the notion of *robustness of combinatorial auctions*, which refers to average efficiency they achieve with respect to different bidding strategies. The fundamental question of this paper is: *"How robust are NLPPAs against non-straightforward bidding strategies."* Any evidence about the performance of auction designs with non-straightforward bidding strategies will not only be important for practical applications, it should also provide a basis for the development of efficient and robust auction designs in the future.

In the following, we will describe the results of computational experiments analyzing allocative efficiency, revenue distribution, ablity to address the threshold problem, and speed of convergence of different NLPPAs against those of the VCG auction, ALPS (Approximative Linear PriceS), and the Combinatorial Clock auction, two designs that use linear prices based on different value models and different bidding strategies. We have decided to run our experiments without activity rules, because several rules have been discussed and using a specific one will distort the comparison. The often used monotonicity rule for example makes it impossible to follow a straightforward bidding strategy.

The paper is structured as follows. In section 2, we will briefly summarize the economic environment, the auction designs in question, and the performance metrics that we use. Section 3 will describe the setup of our computational experiments. In section 4, we will summarize the results and then provide conclusions in section 5.

2. Related Theory and Auction Formats

This section provides an overview of iterative combinatorial auctions and the relevant theory so that the paper is self-contained. We refer the reader to Parkes (2006) for a more detailed introduction to ICAs.

2.1. Winner Determination and Pricing

The typical bidding process in an ICA consists of the steps of bid submission, bid evaluation (aka winner determination, market clearing, or resource allocation) followed by feedback to the bidders. The feedback is typically given in form of ask prices and the provisionall allocation.

Let $\mathcal{K} = \{1, \ldots, m\}$ denote the set of items indexed by k and $\mathcal{I} = \{1, \ldots, n\}$ denote the set of bidders indexed by i with private valuations $v_i(S) \geq 0$, $v_i(\emptyset) = 0$ for bundles $S \subseteq \mathcal{K}$. In addition we assume free disposal: If $S \subset T$ then $v_i(S) \leq v_i(T)$.

Given the private bidder valuations for all possible bundles, the efficient allocation can be found by solving the ***Winner Determination Problem (WDP)***. WDP can be formulated as a binary program using the decision variables $x_i(S)$ which indicate whether the bid of the bidder i for the bundle S belongs to the allocation:

$$\begin{aligned}
&\max_{x_i(S)} \sum_{S \subseteq \mathcal{K}} \sum_{i \in \mathcal{I}} x_i(S) v_i(S) \\
&\text{s.t.} \\
&\sum_{S \subseteq \mathcal{K}} x_i(S) \leq 1 \qquad \forall i \in \mathcal{I} \qquad \textbf{(WDP)} \\
&\sum_{S: k \in S} \sum_{i \in \mathcal{I}} x_i(S) \leq 1 \qquad \forall k \in \mathcal{K} \\
&x_i(S) \in \{0, 1\} \qquad \forall i, S
\end{aligned}$$

The first set of constraints guarantees that any bidder can win at most one bundle, which is only relevant for the XOR bidding language. The XOR language is used because it is fully expressive compared to the OR language which allows a bidder to win more than one bid. Subadditive valuations, where a bundle is worth less than the sum of individual items, cannot be expressed using the OR bidding language. The second set of constraints ensures that each item is only allocated once. Much research has focused on solving the winner determination problem, which is known to be NP-hard (Rothkopf et al. 1998, Sandholm 1999, Park and Rothkopf 2005).

Having determined the winning bids, the auctioneer needs to decide what the winners should pay. A simple approach is for bidders to pay the amount of their bids. However, this creates incentives for bidders to shade their bids and might ultimatively lead to strategic complexity, i.e., to speculation and inefficient allocations.

2.2. Vickrey Prices and Competitive Equilibrium Prices

The VCG auction is a generalization of the Vickrey auction for multiple heterogeneous goods. In this auction bidders have a dominant strategy of reporting their true valuations $v_i(S)$ on all bundles S to the auctioneer, who then determines the allocation and respective Vickrey prices. The VCG design charges the bidders the opportunity costs of the items they win, rather than their bid prices.

Although it has a simple dominant strategy, VCG design suffers from a number of practical problems since its outcome can be outside of the *core* (Rothkopf 2007, Ausubel and Milgrom 2006b).

Formally, let N denote the set of all bidders $\mathcal{I}$ and the auctioneer with $i \in N$, and $M \subseteq N$ be a coalition of bidders with the auctioneer. Let $w(M)$ denote the coalitional value for a subset M, equal to the value of the WDP with all bidders $i \in M$ involved. (N, w) is the coalitional game derived from trade between the seller and bidders. Core payoffs π are then defined as follows

$$Core(N, w) = \{\pi \geq 0 | \sum_{i \in N} \pi_i = w(N), \sum_{i \in M} \pi_i \geq w(M) \quad \forall M \subset N\}$$

This means, there should be no coalition $M \subset N$, which can make a counteroffer that leaves themselves and the seller at least as well as the currently winning coalition. In their seminal paper, Bikhchandani and Ostroy (2002) show that there is an equivalence between the core of the coalitional game and the competitive equilibrium for single-sided auctions.

Definition 1 (Competitive Equilibrium, CE (Parkes 2006))**.** *Prices $\mathcal{P}$, and allocation X^* are in competitive equilibrium if allocation X^* maximizes the payoff of every* *bidder and the auctioneer revenue given prices $\mathcal{P}$. The allocation X^* is said to be supported by prices $\mathcal{P}$ in CE.*

Bikhchandani and Ostroy (2002) show that X^* is supported in CE by some set of prices $\mathcal{P}$ if and only if X^* is an efficient allocation. Although CE always exist, they possibly require non-linear and non-anonymous prices. Prices are *linear* if the price of a bundle is equal to the sum of prices of its items, and prices are *anonymous* if prices are equal for every bidder. Non-anonymous ask prices are also called *personalized* prices. Following three types of ask prices are usually discussed:

1. a set of linear anonymous prices $\mathcal{P} = \{p(k)\}$
2. a set of non-linear anonymous prices $\mathcal{P} = \{p(S)\}$
3. a set of non-linear personalized prices $\mathcal{P} = \{p_i(S)\}$

Generating minimal CE prices is desirable, since it usually imposes incentive compatibility of the auction design.

Definition 2 (Minimal CE Prices (Parkes 2006))**.** *Minimal CE Prices minimize the auctioneer revenue $\Pi(X^*, \mathcal{P})$ on an efficient allocation X^* across all CE prices which support it.*

Minimal CE prices are not necessarily unique. One way to derive minimal CE prices could be to use the duals of the WDP. Unfortunately, WDP is a binary program, and the solution will not be integral in most cases if solved as an LP relaxation. Therefore, the duals will not be minimal and over-estimate the value of the items.

2.3. Non-Linear Personalized Price Auctions

Personalized non-linear CE prices can be derived from the dual strengthened problem as described in (Bikhchandani and Ostroy 2002). We will refer to this formulation as NLPPA WDP.

From duality theory follows that the *complementary slackness conditions* must hold in the case of optimality. It has been shown that these are equal to the CE conditions, stating that every buyer receives a bundle out of his demand set $D_i(\mathcal{P})$ and the auctioneer selects the revenue maximizing allocation.

Definition 3 (Demand Set)**.** *The demand set $D_i(\mathcal{P})$ of a bidder i includes all bundles which maximize a bidder's payoff π_i at the given prices $\mathcal{P}$:*

$$D_i(\mathcal{P}) = \{S : \pi_i(S, \mathcal{P}) \geq \max_{T \subseteq \mathcal{K}} \pi_i(T, \mathcal{P}), \pi_i(S, \mathcal{P}) \geq 0, S \subseteq \mathcal{K}\}$$

Complementary slackness provides us with an optimality condition, which also serves as a termination rule for NLPPAs. If bidders follow the straightforward strategy then terminating the auction when each active bidder receives a bundle in a revenue-maximizing allocation will result in the efficient outcome. Note that a demand set can include the empty bundle. Additionally, the starting prices must represent a feasible dual solution (de Vries et al. 2007). A trivial solution is to use zero prices for all bundles.

Individual NLPPA formats have different rules of selecting bundles and bidders whose prices are increased. The Ascending Proxy Auction has been suggested in the context of the US FCC spectrum auction design (Ausubel and Milgrom 2006a, Ausubel et al. 2006). It is similar to iBundle(3), but the use of proxy agents is mandatory, which essentially leada to a sealed-bid auction format. Since this is the main difference compared to iBundle(3), we will only discuss iBundle in the following.

The ***iBundle auction*** calculates a provisional revenue maximizing allocation at the end of every round and increases the prices based on the bids of non-winning (unhappy) bidders. Parkes and Ungar (2000) suggest three different versions of iBundle: iBundle(2) with anonymous prices, iBundle(3) with personalized prices, and iBundle(d) wich starts with anonymous prices and switches to personalized prices for agents which submit bids for disjunct bundles. We will restrict our analysis to *iBundle(2)* and *iBundle(3)* for the questions of this paper.

The ***dVSV auction*** (de Vries et al. 2007) design differs from iBundle in that it does not compute a provisional allocation in every round but increases prices for a *minimally undersupplied set of bidders.*

Definition 4 (Minimally undersupplied set of bidders)**.** *A set of bidders is minimally undersupplied if: In no efficient allocation each bidders receive a bundle from his demand set. And removing one of the bidders forfeits this property.*

Similar to iBundle(3), it maintains non-linear personalized prices and increases the prices for all agents in a minimally undersupplied set based on their bids of the last round.

2.4. Ascending Vickrey Auctions

iBundle(3) and dVSV result in minimal CE prices. Minimal CE prices and VCG payments typically differ. Bikhchandani and Ostroy (2002) show that the *bidders are substitutes condition (BSC)* is necessary and sufficient to support VCG payments in competitive equilibrium.

Definition 5 (Bidders are Substitutes Condition, BSC)**.** *The BSC condition requires*

$$w(N) - w(N \setminus M) \geq \sum_{i \in M} [w(N) - w(N \setminus i)], \forall M \subseteq N$$

If BSC fails, the VCG payments are not supported in any price equilibrium and truthful bidding is not an equilibrium strategy. A bidder's payment in the VCG mechanism is always less than or equal to the payment by a bidder at any CE price. Even though the BSC condition is sufficient for VCG prices to be supported in CE, Ausubel and Milgrom (2006a) show that a slightly stronger *bidder submodularity condition (BSM)* is required for an ascending auction to implement VCG payments.

Definition 6 (Bidder Submodularity Condition, BSM)**.** *BSM requires that for all* $M \subseteq M' \subseteq N$ *and all* $i \in N$ *there is*

$$w(M \cup \{i\}) - w(M) \geq w(M' \cup \{i\}) - w(M')$$

de Vries et al. (2007) show that under BSM their primal-dual auction yields VCG payments. When the BSM condition does not hold, the property brakes down and a straightforward strategy is likely to lead a bidder to pay more than the VCG price for the winning bundle (Dunford et al. 2007). de Vries et al. (2007) also show that when at least one bidder has a non-substitutes valuation an ascending CA cannot implement the VCG outcome. BSM is often not given in realistic value models as the ones provided by CATS (Leyton-Brown et al. 2000).

The ***CreditDebit Auction*** by Mishra and Parkes (2007) is an extension to the dVSV design which achieves the VCG outcome for general valuations. It introduces the concept of universal competitive equilibrium (UCE) prices, which are CE prices for the main economy as well as for every marginal economy, where a single buyer is excluded. Ther auction terminates as soon as UCE prices are reached and VCG payments are determined either as one-time discounts dynamically during the auction.The authors show that truthful bidding is an ex post Nash equilibrium in these auctions. This is not a contradiction with the previous paragraph, since the bidders do not always pay their bids but can receive discounts. The auction, however, shares the central problem of the VCG auction: if buyer submodularity is not given, the outcomes might not be in the core.

2.5. Linear Price Auctions

The ***Combinatorial Clock Auction (CC auction)*** described in Porter et al. (2003) utilizes anonymous linear prices called *item clock prices*. In each round bidders express the quantities desired on the bundles at the current prices. As long as demand exceeds supply for at least one item (each item is counted only once for each bidder) the price clock "ticks" upwards for those items (the item prices are increased by a fixed price increment), and the auction moves on to the next round. If there is no excess demand and no excess supply, the items are allocated corresponding to the last round bids and the auction terminates. If there is no excess demand but there is excess supply (all active bidders on some item did not resubmit their bids in the last round), the auctioneer solves the winner determination problem considering all bids submitted during the auction runtime. If the computed allocation does not displace any bids from the last round, the auction terminates with this allocation, otherwise the prices of the respective items are increased and the auction continues.

The ***ALPS*** (Approximate Linear PriceS) design (Bichler et al. 2009) is based on the *Resource Allocation Design (RAD)* proposed by Kwasnica et al. (2005) and uses anonymous linear ask prices which are derived from the restricted dual of the LP relaxation of the WDP (Rassenti et al. 1982). The termination rule of the RAD auction has been improved in ALPS to prevent premature auction termination. Furthermore, the ask price calculation better minimizes and balances the prices. In a modified version of ALPS (***ALPSm***), all bids submitted in an auction remain active throughout the auction. This rule had a significant positive impact on efficiency in experimental evaluations. In the following, we will only consider the results of ALPSm and the Combinatorial Clock auction to provide a comparison to NLPPAs.

3. Setup of Numerical Simulations

Laboratory experiments are very costly and typically restricted to a small number of treatments and sessions. Therefore, the robustness of NLPPAs against different types of bidding behavior can best be analyzed in computational experiments. Our simulation environment consists of three main components. A *value model* defines valuations of all bundles for each bidder. A *bidding agent* implements a bidding strategy adhering to the given value model and to the restrictions of the specific auction design. An *auction processor* implements the auction logic, enforces auction protocol rules, and calculates allocations and ask prices. For the comparison of auction formats, we use a set of *focus variables*, such as allocative efficiency, revenue distribution, and speed of convergence measured by number of auction rounds. We believe that the number of rounds is the most relevant number to represent the auction duration, since the absolute time will heavily depend on computational capacities of bidders and speed of communication. We use *allocative efficiency* (or simply *efficiency*) as a primary measure to benchmark auction designs. It is measured as the ratio of the total valuation of the resulting allocation X to the total valuation of an efficient allocation X^* (Kwasnica et al. 2005):

$$E(X) = \frac{\sum_{i \in \mathcal{I}} v_i(\bigcup_{S \subseteq \mathcal{K}: x_i(S)=1} S)}{\sum_{i \in \mathcal{I}} v_i(\bigcup_{S \subseteq \mathcal{K}: x_i^*(S)=1} S)}$$

The term $\sum_{i \in \mathcal{I}} v_i(\bigcup_{S \subseteq \mathcal{K}: x_i(S)=1} S)$ can be simplified to $\sum_{i \in \mathcal{I}} \sum_{S \subseteq \mathcal{K}} x_i(S) v_i(S)$ in case of the XOR bidding language, since at most one bundle per bidder can be allocated.

Similarly, we measure the *revenue distribution* which shows how the overall economic gain is distributed between the auctioneer and bidders. Given the resulting allocation X and the bid prices $\{b_i(S)\}$, the *auctioneer's revenue share* is measured as the ratio of the auctioneer's income to the total sum of valuations of an efficient allocation X^*:

$$R(X) = \frac{\sum_{S \subseteq \mathcal{K}} \sum_{i \in \mathcal{I}} x_i(S) b_i(S)}{\sum_{i \in \mathcal{I}} v_i(\bigcup_{S \subseteq \mathcal{K}: x_i^*(S)=1} S)}$$

In order to compare different settings, we will sometimes also plot auctioneer revenue as % of the revenue in the VCG outcome. The cumulative bidders' revenue share is $E(X) - R(X)$. Note that it is possible for two auction outcomes with equal efficiency to have significantly different auctioneer revenues. In the following, we will briefly discuss the value models and behavioral assumptions in our bidding agents.

3.1. Value Models ps: before auction formats?

Since there are hardly any real-world CA data sets available, we have based our research on the Combinatorial Auctions Test Suite (CATS) value models that have been widely used for the evaluation of WDP algorithms (Leyton-Brown et al. 2000). In the following, we will describe a *value model* as a function that generates realistic, economically motivated combinatorial valuations on all possible bundles for every bidder.

In the ***Real Estate 3x3*** value model, the items sold in the auction are the real estate lots k, which have valuations v_k drawn from the same normal distribution for each bidder. Adjacency relationships between two pieces of land l and m (e_{lm}) are created randomly for all bidders. Edge weights $r_{lm} \in [0, 1]$ are generated for each bidder and used to determine bundle valuations of adjacent pieces of land:

$$v(S) = (1 + \sum_{e_{lm}: l,m \in S} r_{lm}) \sum_{k \in S} v_k$$

We use the *Real Estate 3x3* value model with 9 lots and normally distributed individual item valuations with a mean of 10 and a variance of 2. There is a 90% probability of a vertical or horizontal edge, and an 80% probability of a diagonal edge. Edge weights have a mean of 0.5 and a variance of 0.3. Unless explicitly stated (Section 4.5), all auction experiments with the Real Estate value model are conducted with 5 bidders.

The ***Transportation*** value model models a nearly planar transportation graph in Cartesian coordinates, where each bidder is interested in securing a path between two randomly selected vertices (cities). The items traded are edges (routes) of the graph. Parameters for the Transportation value model are the number of items (edges) m and graph density ρ, which defines an average number of edges per city, and is used to calculate the number of vertices as $(m * 2)/\rho$. The bidder's valuation for a path is defined by the Euclidean distance between two nodes multiplied by a random number, drawn from a uniform distribution. We consider the Transportation value model with 25 edges, 15 vertices, and 15 bidders. Every bidder had interest in 16 different bundles on average.

The ***Pairwise Synergy*** value model from An et al. (2005) is defined by a set of valuations of individual items $\{v_k\}$ with $k \in \mathcal{K}$ and a matrix of pairwise item synergies $\{syn_{k,l} : k, l \in \mathcal{K}, syn_{k,l} = syn_{l,k}, syn_{k,k} = 0\}$. The valuation of a bundle S is then calculated as

$$v(S) = \sum_{k=1}^{|S|} v_k + \frac{1}{|S|-1} \sum_{k=1}^{|S|} \sum_{l=k+1}^{|S|} syn_{k,l}(v_k + v_l)$$

A synergy value of 0 corresponds to completely independent items, and the synergy value of 1 means that the bundle valuation is twice as high as the sum of the individual item valuations. The model is very generic, as it allows different types of synergistic valuations, but it was also used to model certain types of transportation auctions (An et al. 2005). In this paper, we use the Pairwise Synergy value model with 7 items, where item valuations are drawn for each auction independently from a uniform distribution between 4 and 12. The synergy values are drawn from a uniform distribution between 1.5 and 2.0. We tested lower synergy values, but found little difference in the results. All auctions with Pairwise Synergy value model have 5 bidders.

In the *Real Estate* and *Pairwise Synergy* value models bidders were interested in a maximum bundle size of 3, because in these value models large bundles are always valued over small ones. This is also motivated by real-world observations An et al. (2005). Without this limitation, the auction usually degenerates into a scenario with a single winner for the bundle containing all items. In the *Transportation* value model bidders were not restricted in bundle size.

For the numerical experiments all valuations for all bidders need to be generated and in each round all bundles need to be sorted by payoff. This leads to substantial computation time and puts a practical limit on the size of the value model, which can be simulated. For example, a single CreditDebit auction with the Real Estate value model took up to two hours due to the high number of auction rounds.

3.2. Behavioral Assumptions and Bidding Agents

Theoretical models, as described in Section 2, provide arguments for straightforward bidding in NLPPAs. In an auction with private valuations, however, when the bidders do not know whether the bidder submodularity holds, and therefore can decide to shade their bids. There are also cognitive barriers for the straightforward strategy, since bidders need to determine their demand set for an exponential number of possible bundle bids in each round.

It is useful to look at empirical observations and the behavioral literature to derive hypotheses on bidding strategies in NLPPAs. Unfortunately, so far, there have only been a few lab experiments using NLPPAs and we do not know of any applications in the field. Chen and Takeuchi (2009) compared the VCG auction and iBEA in experiments where humans competed against artificial bidders. Bidders were significantly more likely to bid on packages with a high temporary profit, but did not follow the pure straightforward strategy. More recently, Scheffel et al. (2009) conducted experiments comparing iBundle, ALPS, Combinatorial Clock, and the VCG auction. Again, bidders in the iBundle auction did not follow the straightforward strategy, even though they were provided with a decision support tool that helped them select their demand set. There was, however, a high likelihood to bid on one of the best 10 bundles based on their payoff in the current round. Bidder idiosyncrasies and mixed strategies such as in trembling-hand perfect equilibria (Selten 1975) are possible explanations of these findings.

We have developed a number of *bidding agents* that are motivated by different conjectures about the behavior of bidders in NLPPAs. These agents implement a bidding strategy adhering to the given value model and to the restrictions of a specific auction format.

The ***straightforward*** bidder is motivated by the theory. He always bids his demand set which maximizes his surplus if it were to win one of its bids at the current prices. Obviously, the results of auctions with straightforward bidders shall achieve the outcomes predicted by the theory.

From lab experiments, we know that bidders do not always follow the straightforward strategy due to cognitive reasons and simple errors. The ***forgetful*** agent follows the straightforward strategy, but "forgets" to submit 10% of his bids in each round. These 10% are determined randomly and independently for every round. Similarly we also modeled a ***heuristic*** bidder. This agent randomly selects 5 out of his 20 best bundles based on his payoff in a round.

Sometimes bidders in the lab try to increase their chances of winning by bidding on many bundles, which can be explained by risk aversion. The ***powerset*** bidder evaluates all possible bundles in each round, and submits bids for bundles which are profitable given current prices. We have limited this bidder to 10 bundles with the highest payoff in each round, which is based on the observation that bidders typically do not bid on many bundle bids in an auction round (Scheffel et al. 2009). Typically, in our settings these 10 bundles will have up to 20 % difference in payoff at the beginning of the auction given equal start prices.

The NLPPAs, which we study in this work, terminate as soon as the new provisional allocation includes bids from all active bidders. This termination rule may motivate collusive bidders to submit more then just the demand set in the first round in the hope that a suitable allocation is found early and the auction terminates before the prices rise. The ***level*** bidder models a dishonest strategy that tries to exploit this idea. We modify the straightforward bidder by lowering valuations of his best l bundles and setting all of them equal to the valuation of the l^{th} best bundle. In our simulations, we have used $l = 10$. Note that proxy bidding agents cannot prevent bidders from adopting this strategy.

If a bidder is restricted in time during the auction, he might select his most valuable bundles a priori, and stick to this selection throughout the auction. This ***preselect*** agent selects his 20 most valuable bundles and follows the straightforward strategy, only using the preselected bundles. Again, a proxy cannot detect or prevent this strategy.

To ensure comparability between auction formats, we use a fixed minimum increment of 1 and integer valuations for all valuations. Under these conditions, NLPPAs always terminate with an efficient solution given straightforward bidding (de Vries et al. 2007).

4. Results

This section describes the results of simulations in terms of efficiency, revenue distribution and the number of auction rounds. Unless explicitly stated otherwise, each auction setup was repeated 50 times with different random seeds for value models and, where appropriate, bidding agents. Overall, we provide the results of an extensive experimental design with more than 25,000 auctions.

Bidding Agent \ ICA Format		CreditDebit	dVSV	iBundle(3)	iBundle(2)	ALPSm	Clock	VCG (tr)
Straightforward	Efficiency in %	100.00	100.00	100.00	99.90	98.64	95.16	100.00
	Rev. Auctioneer in %	83.45	84.57	84.61	83.88	84.09	87.05	83.17
	Rev. all bidders in %	16.55	15.43	15.39	16.01	14.55	8.10	16.83
	Rounds	139.96	137.90	151.28	149.26	72.56	28.76	1.00
Forgetful 10% chance to forget each bid	Efficiency in %	99.92	99.78	100.00	99.92	98.64	96.35	
	Rev. Auctioneer in %	81.73	85.28	84.73	83.88	84.11	88.02	
	Rev. all bidders in %	18.19	14.51	15.27	16.03	14.54	8.30	
	Rounds	550.54	545.30	329.68	237.20	72.26	29.52	
Level10	Efficiency in %	90.00	90.05	89.71	90.12	91.12	86.67	
	Rev. Auctioneer in %	72.10	72.29	72.34	71.55	72.25	76.46	
	Rev. all bidders in %	18.12	17.97	17.58	18.72	19.01	10.30	
	Rounds	82.38	82.00	133.22	130.96	128.88	26.06	
Powerset10 10 best bundles selected in each round	Efficiency in %	90.50	89.67	98.48	99.20	99.57	97.27	
	Rev. Auctioneer in %	23.10	71.53	82.93	82.14	87.65	94.09	
	Rev. all bidders in %	67.33	18.22	15.55	17.02	11.91	3.20	
	Rounds	1525.44	1519.58	979.18	283.46	24.94	25.30	
Preselect20 20 best bundles preselected before the auction	Efficiency in %	98.24	98.24	98.24	97.56	91.51	92.07	
	Rev. Auctioneer in %	76.26	79.55	79.27	78.69	76.35	84.65	
	Rev. all bidders in %	21.94	18.64	18.92	18.79	15.07	7.39	
	Rounds	147.18	141.32	146.34	145.62	61.76	28.32	
Heuristic 5 random bundles out of 20 best	Efficiency in %	76.09	73.68	98.94	97.51	99.29	98.18	
	Rev. Auctioneer in %	22.11	52.16	82.03	83.55	87.88	94.19	
	Rev. all bidders in %	54.07	21.56	16.88	14.02	11.40	4.00	
	Rounds	3183.12	3058.88	1860.88	524.04	27.44	25.60	

Table 1: Performance of ICAs with differing Bidding Agents for the Real Estate value model

4.1. Analysis of Different Bidding Strategies

First we assume that all bidders in the auction use the same strategy and analyze the results for different value models and bidding strategies. In particular, we want to investigate how different NLPPAs behave when the bidders deviate from the straightforward strategy.

The average performance metrics are summarized in Table 1 for the Real Estate, in Table 2 for the Pairwise Synergy, and in Table 3 for the Transportation value models.

Interestingly, we found a similar pattern in the results for all value models. We have also repeated the same tests with different instances of the Pairwise Synergy value model, where the synergy level was lower and in some cases negative (subadditive valuations), and with the Real Estate value model with a maximum bundle size of four instead of three. These modifications led to similar results.

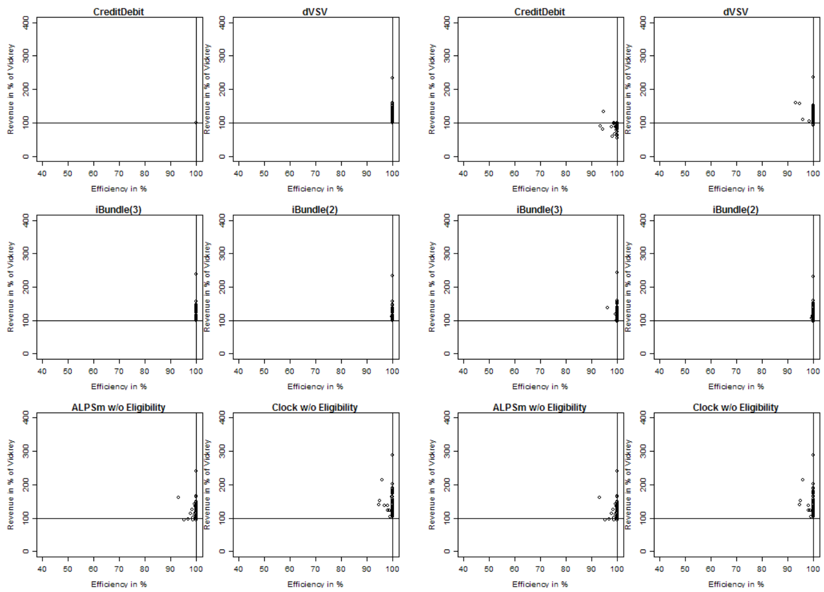
Only the Transportation value model was different with respect to its high robustness against preselect bidding. The main reason is the low number of bundles with significant competition, which is due to the underlying topology of transportation networks and the fact that only a few bundles are of interest to every bidder.

Bidding Agent \ ICA Format		CreditDebit	dVSV	iBundle(3)	iBundle(2)	ALPSm	Clock	VCG (tr)
Straightforward	Efficiency in %	100.00	100.00	100.00	100.00	99.65	99.53	100.00
	Rev. Auctioneer in %	89.63	90.53	90.46	90.47	89.81	92.83	89.60
	Rev. all bidders in %	10.37	9.47	9.54	9.53	9.85	6.69	10.40
	Rounds	204.44	202.96	154.96	154.82	68.70	31.68	1.00
Forgetful	Efficiency in %	99.81	99.65	100.00	99.99	99.65	99.56	
10% chance to	Rev. Auctioneer in %	88.31	91.04	90.41	90.54	89.92	92.97	
forget each bid	Rev. all bidders in %	11.50	8.63	9.59	9.45	9.73	6.58	
	Rounds	715.40	712.26	333.48	243.32	68.60	31.86	
Level10	Efficiency in %	98.40	98.41	98.48	98.47	97.21	94.46	
	Rev. Auctioneer in %	88.38	88.78	88.82	88.80	88.08	86.90	
	Rev. all bidders in %	10.03	9.64	9.67	9.68	9.14	7.59	
	Rounds	150.16	149.44	150.28	150.10	117.74	33.66	
Powerset10	Efficiency in %	96.01	95.91	99.16	99.55	99.75	99.51	
10 best bundles	Rev. Auctioneer in %	35.82	87.06	89.01	89.98	92.28	98.19	
selected	Rev. all bidders in %	60.18	8.85	10.15	9.57	7.47	1.31	
in each round	Rounds	1353.50	1352.36	650.24	192.60	29.34	31.24	
Preselect20	Efficiency in %	85.80	85.80	85.80	85.80	82.75	85.21	
20 best bundles	Rev. Auctioneer in %	79.47	79.91	79.97	79.98	77.07	82.30	
preselected	Rev. all bidders in %	6.35	5.90	5.84	5.84	5.70	2.91	
before the auction	Rounds	230.90	230.06	138.54	138.46	48.14	30.56	
Heuristic	Efficiency in %	85.33	85.40	98.70	98.15	99.40	99.35	
5 random bundles	Rev. Auctioneer in %	25.97	60.64	88.86	88.85	92.87	97.92	
out of 20 best	Rev. all bidders in %	59.37	24.78	9.84	9.30	6.53	1.41	
	Rounds	2551.94	2544.98	1261.50	338.50	31.86	31.34	

Table 2: Performance of ICAs with differing Bidding Agents for the Pairwise Synergy value model

Bidding Agent \ ICA Format		CreditDebit	dVSV	iBundle(3)	iBundle(2)	ALPSm	Clock	VCG (tr)
Straightforward	Efficiency in %	100.00	100.00	100.00	99.99	99.55	99.48	100.00
	Rev. Auctioneer in %	56.13	66.93	65.43	65.36	65.92	77.10	55.49
	Rev. all bidders in %	43.87	33.07	34.57	34.63	33.60	22.41	44.51
	Rounds	216.78	205.18	78.66	78.02	32.24	29.64	1.00
Forgetful	Efficiency in %	99.47	99.60	99.93	99.97	99.55	99.58	
10% chance to	Rev. Auctioneer in %	51.38	67.88	65.55	65.81	65.92	77.10	
forget each bid	Rev. all bidders in %	48.10	31.76	34.35	34.15	33.60	22.48	
	Rounds	434.96	414.70	124.84	106.90	32.24	29.74	
Level10	Efficiency in %	84.95	85.06	83.64	84.01	83.96	84.56	
	Rev. Auctioneer in %	23.13	26.56	26.36	26.30	27.65	37.05	
	Rev. all bidders in %	61.92	58.46	57.31	57.70	56.19	47.36	
	Rounds	60.56	56.06	40.14	39.38	19.62	14.02	
Powerset10	Efficiency in %	91.33	91.73	97.28	97.26	99.78	97.39	
10 best bundles	Rev. Auctioneer in %	2.47	56.09	58.94	59.95	72.56	88.60	
selected	Rev. all bidders in %	88.80	35.52	38.19	37.20	27.18	8.82	
in each round	Rounds	312.56	311.36	154.06	93.36	19.80	25.00	
Preselect20	Efficiency in %	99.80	99.80	99.80	99.80	99.52	99.24	
20 best bundles	Rev. Auctioneer in %	55.40	66.51	65.24	65.24	66.44	76.77	
preselected	Rev. all bidders in %	44.39	33.28	34.56	34.56	33.06	22.50	
before the auction	Rounds	217.24	205.62	78.64	78.12	32.56	29.88	
Heuristic	Efficiency in %	84.14	84.50	96.24	96.10	99.75	97.73	
5 random bundles	Rev. Auctioneer in %	6.06	54.98	59.04	59.59	73.82	88.35	
out of 20 best	Rev. all bidders in %	78.26	29.75	37.10	36.43	25.90	9.33	
	Rounds	789.24	788.24	268.62	158.12	21.20	25.08	

Table 3: Performance of ICAs with differing Bidding Agents for the Transportation value model



(a) Straightforward (b) Forgetful

Figure 1: Efficiency and revenue for the Transportation value model

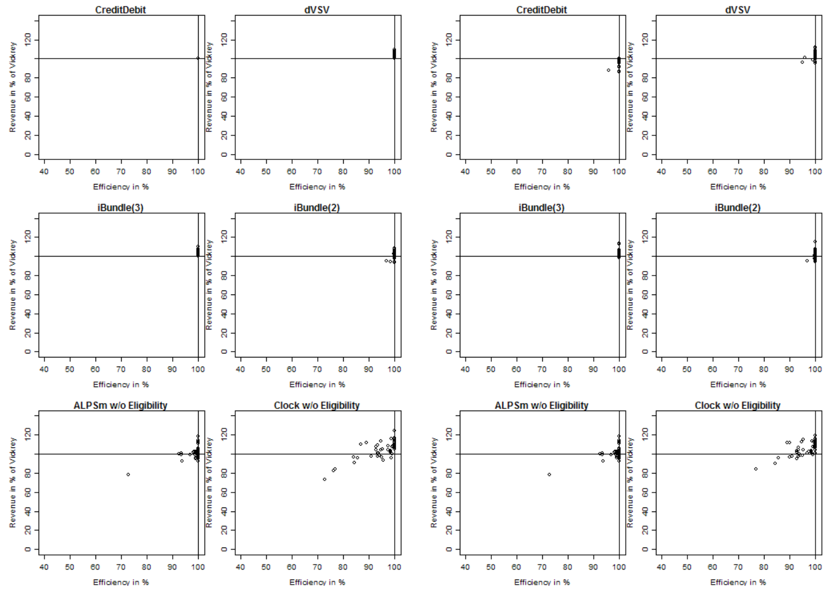
For the same reason, the level bidders could successfully collude and significantly increase their payoff.

Below we describe the main findings for every type of the bidding strategy, for the case when all bidders in the auction follow it.

Straightforward Bidder. Our computational experiments with straightforward bidders and NLPPAs yielded outcomes in line with the theory. All NLPPAs were efficient. iBundle(3) and dVSV achieved VCG outcomes only when the BSC condition was satisfied. When BSC was not satisfied, iBundle(3) and dVSV resulted in higher prices. In the Transportation value model we could observe cases where the prices in NLPPAs were up to 250% higher than in the VCG auction (see Figure 1), whereas in the Real Estate and Pairwise Synergy value models, the price increase was low (see Figures 2 and 3).

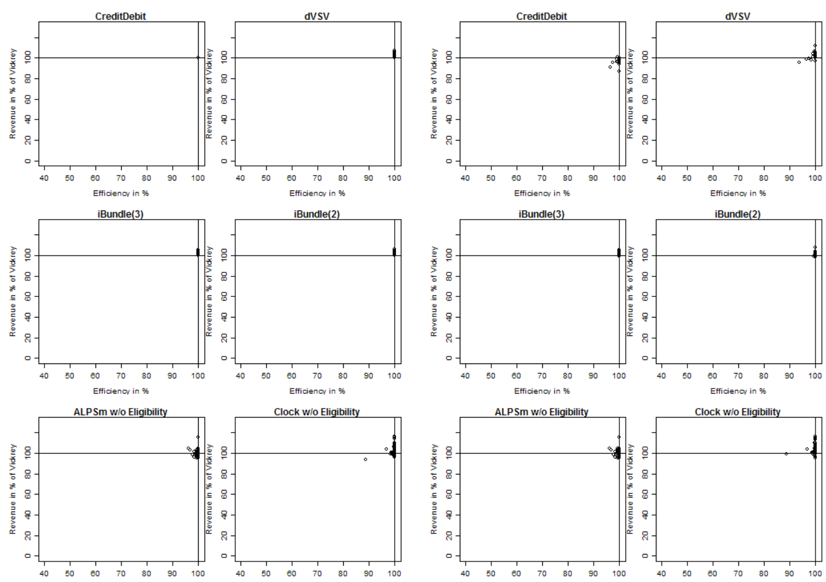
The CreditDebit auction always resulted in VCG payoffs, as theory predicts. iBundle(2) did not result in an efficient outcome for some instances, but these occasions were rare and the loss of efficiency generally very low. One reason is, that iBundle(2) is efficient for superadditive valuations (Parkes and Ungar 2000), which is mostly the case in our value models.

In the linear-price auctions (ALPS and CC) the straightforward bidder was less



(a) Straightforward (b) Forgetful

Figure 2: Efficiency and revenue for the Real Estate 3x3 value model



(a) Straightforward (b) Forgetful

Figure 3: Efficiency and revenue for the Pairwise Synergy value model

efficient than NLPPAs. Still, their efficiency was 95.16% and 98.64% on average for the Real Estate value model, and even more than 99% for the Pairwise Synergy and the Transportation value model. It is important to note that with the straightforward bidder, we observed cases in ALPSm and CC auctions, where efficiency was as low as 70%. If bidders follow the straightforward strategy in ALPS and the CC auction, it can happen that they do not reveal certain valuations that would be part of the efficient solution (Bichler et al. 2009). In the presence of activity rules, bidders are forced to bid on more than just their demand set. This will have a positive effect on the robustness of ALPSm format as we will see when we discuss powerset bidders.

Forgetful Bidder. NLPPAs were fairly robust against forgetful bidders, and the efficiency losses were low. Only the number of auction rounds increased significantly across all value models. For example, the CreditDebit auction took on average 139.96 auction rounds in the Real Estate value model with straightforward bidders and 550.54 rounds with forgetful bidders. Interestingly, the linear price auctions were hardly impacted at all compared to the straightforward bidding strategy. Also the average number of auction rounds remained almost the same.

Level Bidder. As expected, the collusive level bidder causes an efficiency loss. In the Real Estate value model efficiency dropped to around 90% in all auction formats and also the auctioneer revenue was not significantly different (t-test, $\alpha = 0.05$). In the Transportation value model, this strategy was very successful. Here, the level bidder achieved a significantly higher revenue than with a straightforward strategy, however, at the expense of efficiency, which dropped to around 84% on average. For the Transportation value model, the competition is focused on a small number of items or legs in the transportation network and it is more likely that a valid allocation is found earlier when all bidders follow the level bidding strategy.

In the CreditDebit auction the high bidder revenue caused by overestimated price discounts due to the bid shading in the level strategy. In all auction formats, however, there are also instances in which the auctioneer gained more and the bidders gained less compared to the straightforward bidding strategy. This strategy works only in an expected sense if all bidders adhere to it. It does not represent a stable equilibrium.

Powerset Bidder. For the iBundle(2), iBundle(3), dVSV, and the CreditDebit auction, this strategy led to a significant decrease in efficiency (t-test, $\alpha = 0.05$). For iBundle auctions the efficiency loss was lower than for dVSV and the CreditDebit auctions. Apparently, the price calculation algorithm using minimally undersupplied set is less robust against non-straightforward bidding.

In contrast, the efficiency and auctioneer revenue share of ALPS auctions was equal or higher compared to the straightforward strategy in all value models. Simultaneously the number of rounds was significantly reduced. The CC auction performed well in homogenous markets, modeled by Real Estate and Pairwise Synergy value models. Typically, these linear-price auctions are used with strong activity rules to encourage the revelation of many bundle preferences already in the early rounds of an auction, which might lead to a similar strategy with bidders in the field.

Preselect Bidder. The efficient solution cannot be found if it includes the bundles which are omitted by the preselect bidder. In the Transportation value model this had little effect on efficiency compared to straightforward bidding, since there is only a small number of interesting bundles for every bidder. In other value models we could see a significant decrease in all measurements.

Heuristic Bidder. Heuristic bidder, who bids on ramdom 5 of his 20 best bundles, causes significant efficiency losses in all NLPPAs. We observed the highest efficiency losses for dVSV and the CreditDebit auction. The reason for the low revenue of the CreditDebit auction is that discounts are miscalculated if not all bundle bids are available at the end. In addition, the more complex price update rule is less robust against non-straightforward bidding.

4.2. Sensitivity wrt. Straightforward Bidding

We have conducted another set of auctions using Real Estate and Transportation value models to measure the effect of one single bidder deviating from the straightforward strategy, while the rest adheres to it. For each setting, we run 50 auctions using iBundle(3), iBundle(2) and ALPSm formats.

The results follow the same pattern over all three ICA formats and both value models. The allocative efficiency was not impacted, except that already a single level bidder reduced the efficiency significantly. The level bidder also suffered highest revenue loss of 46% of his revenue, followed by the preselect bider, who had just a minor loss of less then 5%. All other bidder types did not change the auction outcome significantly. This indicates that the equilibrium, which brings significant increase in revenue to level bidders when all bidders follow this strategy, is not stable.

4.3. Threshold problem

The threshold problem is characterized by a situation where several local bidders are competing against a global bidder. The local bidders are interested in individual lots or small bundles, and the global bidder tries to win a larger set of lots. A successful auction design shall help the local bidders to coordinate their bids in order to win against the global bidder in case such an allocation is efficient.

We analyzed different ICA designs with respect to the threshold problem using the Real Estate value model with dedicated local bidder for each of the nine lots and two global bidders. The valuations were selected such that they impose a high competition between local and global bidders. In 20 selected instances the efficient allocation included the nine local bidders and not the global bidders and the allocation with global bidders was within 10% from the optimal solution. We focused on the straightforward, the forgetful, the heuristic, and the powerset10 bidder.

Bidding Agent \ ICA Format	CreditDebit	dVSV	iBundle(3)	iBundle(2)	ALPSm	Clock	VCG
Straightforward	20	20	20	18	19	20	20
Forgetful	1	1	6	5	19	18	
Powerset10	1	1	4	2	19	19	
Heuristic	0	0	4	4	20	20	

Table 4: Number of instances of the threshold problem, where small bidders actually won

As Table 4 illustrates, both linear and non-linear auction formats can solve the threshold problem with straightforward bidding. If the bidders deviated from straightforward bidding, the NLPPAs failed to solve the threshold problem in most cases. The reason is that the NLPPAs do not preserve information about bids submitted in previous rounds. Only straightforward bidders always include old bids with updated prices since the demand set can only grow throughout the auction given the NLPPA price update rule. Strong activity rules, as suggested by Mishra and Parkes (2007), can be used to force bidders to conform to the straightforward bidding strategy and improve performance of NLPPAs in this situation. However such strong rules require the bidder to evaluate and rank all bundles in advance and virtually transform the auction in to a sealed-bid format, thus eliminating the advantages of an iterative preference elicitation process.

4.4. Speed of Convergence

The CC auction had the lowest number of auction rounds in all treatments. On average, NLPPAs took three times as many rounds as linear-price based auctions (Figure 4). In contrast to the theory that expects a lower number of auction rounds in dVSV compared to iBundle(3), we observed a higher number of rounds in dVSV. This happens because the minimally undersupplied set is not unique and we used the smallest possible minimally undersupplied set we found, which resulted in small price steps. For the same reason, non-straightforward strategies caused the highest increase in rounds for dVSV and CreditDebit auctions, compared to other formats. The speed of convergence of these two formats can be improved by increasing prices on several disjunct minimally undersupplied sets in every round.

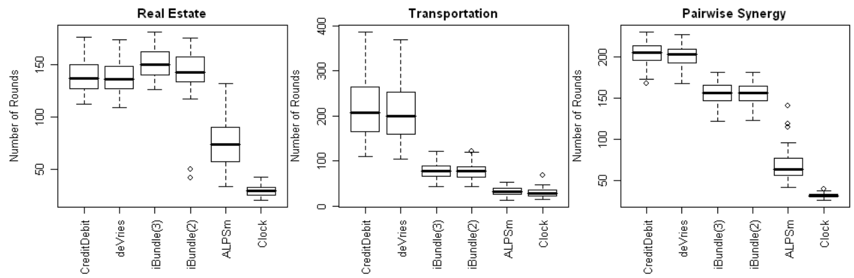


Figure 4: Auction rounds

4.5. Impact of Increasing Competition

Auctions are expected to yield more revenue if there is more competition. Table 5 presents results of different auction formats using the Real Estate value model and a varying number of bidders. Each number represents an average of 10 auctions with same setting and different random seeds for the value model. We observed the expected behavior in NLPPAs and in the ALPSm design. The average revenue share in the CC auctions decreased from 5 to 7 bidders. Linear price-based auctions and the iBundle design showed a lower number of rounds with an increasing number of bidders. In contrast, the dVSV and CreditDebit auctions showed a massive increase of rounds. This is explained by different price update mechanisms. The iBundle design, which increases prices for all unhappy bidders, will generally increase more prices when the competition is higher. The dVSV and CreditDebit auctions, which increase prices for a minimally undersupplied set of bidders, will be able to find a smaller minimally undersupplied set (typically with only one bidder) when the competition increases, and therefore increase prices for less bundles in each round.

4.6. Impact of BSC

We have discussed that if BSM is satisfied, NLPPAs will lead to Vickrey prices. Due to computational reasons, we have restricted ourselves to analyze the somewhat weaker BSC condition only. The results based on the Real Estate value model, where the BSC condition was fulfilled in approximately half of the randomly generated instances, and all agents were following the straightforward strategy are summarized in Tables 6 and 7, as well as Figure 5. As expected, prices and consequently revenue were higher than the VCG outcome in NLPPAs. For linear-price auctions, the impact was not significantly different.

Bidding Agent \ ICA Format		iBundle(2)	iBundle(3)	dVSV	CreditDebit	Clock	ALPS m	VCG (tr)
4 bidders BSC fulfilled 100 %	Efficiency in %	99.74	100.00	100.00	100.00	96.69	95.88	100.00
	Rev. Auctioneer in %	78.49	80.34	80.34	79.96	84.17	73.82	79.96
	Rev. all bidders in %	21.25	19.66	19.66	20.04	12.52	22.06	20.04
	Rounds	195.84	201.46	83.40	83.40	35.66	103.06	1.00
5 bidders BSC fulfilled 90 %	Efficiency in %	99.94	100.00	100.00	100.00	96.52	95.06	100.00
	Rev. Auctioneer in %	84.48	84.94	84.94	83.16	88.09	77.27	83.16
	Rev. all bidders in %	15.46	15.06	15.06	16.84	8.43	17.79	16.84
	Rounds	148.96	150.10	143.72	146.28	31.14	68.92	1.00
6 bidders BSC fulfilled 50 %	Efficiency in %	99.91	100.00	100.00	100.00	94.74	97.06	100.00
	Rev. Auctioneer in %	87.00	87.20	87.39	85.42	88.04	82.44	85.42
	Rev. all bidders in %	12.91	12.80	12.61	14.58	6.69	14.62	14.58
	Rounds	132.58	133.42	207.10	209.62	30.68	61.86	1.00
7 bidders BSC fulfilled 40 %	Efficiency in %	99.89	100.00	100.00	100.00	94.35	96.98	100.00
	Rev. Auctioneer in %	88.29	88.61	88.79	86.38	87.94	84.45	86.38
	Rev. all bidders in %	11.59	11.39	11.21	13.62	6.41	12.54	13.62
	Rounds	122.06	122.82	271.46	274.54	29.92	52.58	1.00

Table 5: Comparison of ICAs with differing competition levels

Revenue \ ICA Format	iBundle(2)	iBundle(3)	dVSV	CreditDebit	Clock	ALPSm
Min	98.86	100.00	100.00	100.00	102.40	86.68
Mean	99.87	100.00	100.00	100.00	107.51	97.25
Max	101.72	100.00	100.00	100.00	120.81	116.51

Table 6: Revenue in % to the VCG outcome, in the Real Estate value model with BSC fulfilled

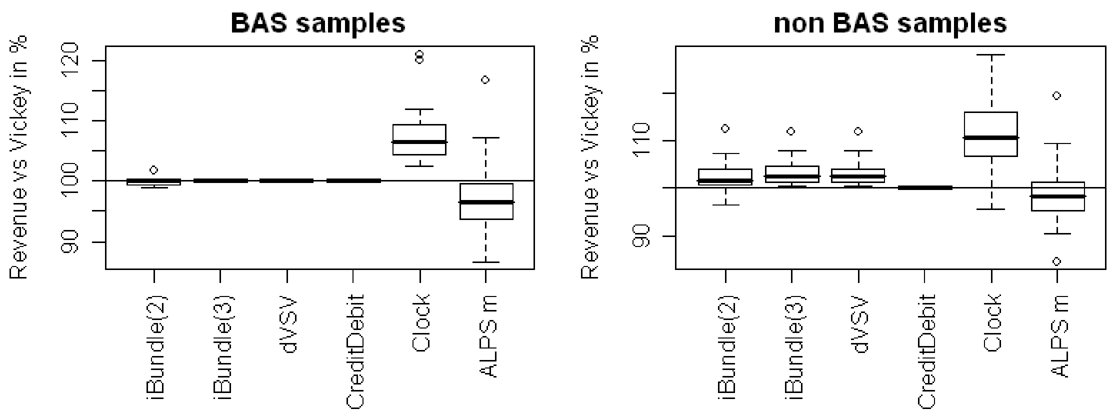


Figure 5: Impact of BSC on prices for straightforward bidding with the Real Estate value model

4.7. Summary

Interestingly, we had similar results regarding efficiency, auctioneer's revenue share, and auction rounds across all value models. Only the Transportation value model was different wrt. the high level of ask prices compared to the VCG auction, and also its stability against preselect bidding. The main reason was the low number of bundles with significant competition that is due to the underlying topology of transportation networks. For the same reason, the level bidders could successfully collude and significantly increase their payoff.

Revenue \ ICA Format	iBundle(2)	iBundle(3)	dVSV	CreditDebit	Clock	ALPSm
Min	96.55	100.52	100.52	100.00	95.71	84.56
Mean	102.48	103.31	103.30	100.00	111.39	98.61
Max	112.34	111.69	111.69	100.00	127.99	119.25

Table 7: Revenue in % to the VCG outcome, in the Real Estate value model with BSC not fulfilled

Linear price auctions were robust against all strategies, except the level strategy, which assumes collusive behaviour and makes it difficult for any auctioneer to select an efficient solution in general. NLPPAs were robust against forgetful bidders, but at the expense of a high number of bidding rounds. There were significant efficiency losses in NLPPAs with heuristic bidders, powerset, level, and preselect bidders.

dVSV and CreditDebit auctions have a significantly lower efficiency than iBundle(3) with heuristic and powerset bidders. The main difference between these formats is the set of bidders, for which the ask prices are increased. Increasing the ask prices on a minimally undersupplied set of bidders is less robust against these strategies. Note that Proxy agents, which can be used to enforce straightforward bidding (Mishra and Parkes 2004), cannot detect and prevent level and preselect bidding strategies.

5. Conclusion

NLPPAs such as iBundle(3), dVSV, iBEA, and CreditDebit auctions have greatly advanced our understanding for the design of efficient auction mechanisms in the realm of private valuations. These formats are modeled after well-known optimization algorithms that lead to efficient solutions, provided that bidders follow the straightforward strategy. Since these are exact algorithms, the auction generally requires many rounds, where all 2^m valuations of all losing bidders are elicited. Both the high number of auction rounds and the necessity of the straightforward bidding strategy motivate the use of proxy agents, which need to be hosted by a trusted third party, which essentially reduces the auction to a sealed-bid event for the bidders. While there are incentives for bidders to follow the straightforward strategy in these auctions, it is not always acceptable to use proxy agents, nor can they prevent certain strategies, such as level bidding.

In contrast, linear price combinatorial auctions follow a more heuristic approach to find the optimal solution. While our results achieve high efficiency values on average, one can easily construct examples, where linear price CAs cannot be efficient (Bichler et al. 2009). There are a few remedies, such as the proxy phase in the Clock-Proxy auction (Ausubel et al. 2006) that addresses these inefficiencies, but these designs have not yet been thoroughly analyzed (Blumrosen and Nisan 2005). Linear-price designs suffer from the fact that they cannot be 100% efficient, but they have shown to be more robust against many strategies and bear a few advantages. The main advantage is the a linear number of prices needed. This reduces the communication between auctioneer and bidders and presents a guideline to help bidders find profitable items and bundles (Kwon et al. 2005). They also exhibit a low number of auction rounds compared to NLPPAs. The perceived fairness of anonymous prices might be an issue in some applications too. Overall, robustness of combinatorial auction formats against different bidding strategies is another important criterion auctioneers need to care about.

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