Efficiency with Linear Prices? A Theoretical and Experimental Analysis of the Combinatorial Clock Auction.

Martin Bichler, Pasha Shabalin, Georg Ziegler
Department of Informatics, TU München, Germany

Combinatorial auctions have been suggested as a means to raise efficiency in multi-item negotiations with complementarities among goods as they can be found in procurement, energy markets, transportation, and the sale of spectrum auctions. The Combinatorial Clock (CC) auction has become very popular in these markets for its simplicity and for its highly usable price discovery, derived by the use of linear prices. Unfortunately, no equilibrium bidding strategies are known. Given the importance of the CC auction in the field, it is highly desirable to understand whether there are efficient versions of the CC auction, providing a strong game theoretical solution concept. So far, equilibrium strategies have only been found for combinatorial auctions with non-linear and personalized prices for very restricted sets of bidder valuations. We provide an extension of the CC auction, the CC+ auction, and show that it actually leads to efficient outcomes in an ex-post equilibrium for general valuations with only linear ask prices. We also provide a theoretical analysis on the worst case efficiency of the CC auction, which highlights problems in the valuations, in which the CC is very inefficient. As in all other theoretical models of combinatorial auctions, bidders in the field might not be able to follow the equilibrium strategies suggested by the game-theoretical predictions. Therefore, we complement the theoretical findings with results from computational experiments using realistic value models. This analysis helps to understand the impact of deviations from the equilibrium strategy and the robustness of such auctions. The experimental analysis shows that the CC auction and its extensions have a number of virtues in practical applications, in particular a low number of auction rounds and bids submitted compared to auction designs with non-linear and personalized ask prices.

Key words: electronic markets and auctions, combinatorial clock auction, allocative efficiency, core-selecting auctions

---

Version October 13, 2010

1. Introduction

The development of the Internet allowed for the exchange of complex preference profiles and laid the foundation for the design of new market mechanisms. The promise of these mechanisms is that by allowing market participants to reveal more comprehensive information about cost structures or utility functions, they can increase allocative efficiency and lead to higher economic welfare. In recent years, a growing body of literature in the Management Sciences is devoted to the design of such smart markets (McCabe et al. 1991, Gallien and Wein 2005), with combinatorial auctions (CAs) emerging as a pivotal example (Cramton et al. 2006b). In CAs, multiple items are sold simultaneously and they allow bids on packages of items. Nowadays, CAs are being used for the sale of spectrum licenses in Europe and the US (Cramton 2009), for transportation (Caplice 2006), and in industrial procurement (Bichler et al. 2006, Sandholm and Begg 2006). Much recent research in Information Systems is devoted to the design and analysis of CAs and respective decision support tools (Adomavicius and Gupta 2005, Xia et al. 2004, Bapna et al. 2007, Guo et al. 2007, Bichler et al. 2009, Scheffel et al. 2010). A summary of recent and emerging research in IS can be found in Bichler et al. (2010).

Although CAs yield higher levels of efficiency in the lab in the case of complementarities compared to simultaneous auctions without package bids, equilibrium strategies are unknown for many

CA formats used in the field. The exponential number of possible package bids leads to high strategic complexity for bidders and the bidding strategies observed in the lab are diverse, with some bidders bidding on many and others bidding on only a few packages of interest in each round (Goeree and Holt 2008, Scheffel et al. 2010). Finding efficient combinatorial auction designs which satisfy a strong game-theoretical solution concept can help reduce the strategic complexity and lead to higher efficiency as a result. Even if the assumption for such equilibrium strategies is not given in particular applications, it helps to understand the sources of inefficiency observed in the lab or in the field.

Green and Laffont (1977) already proved that an efficient mechanism in which honest revelation is a dominant strategy for each agent is necessarily a Vickrey-Clarke-Groves (VCG) mechanism. While this initially appears to be the silver bullet for the design of combinatorial auctions, VCG mechanisms turned out to be impractical in most applications (Ausubel and Milgrom 2006b, Rothkopf 2007). For single-item auctions, not only the Vickrey auction, but also the ascending Clock auction (aka Japanese auction) is individually rational, efficient, and has a dominant strategy (i.e., it is strategy-proof). Iterative combinatorial auction formats help market participants reduce the number of interesting packages through price feedback throughout the auction. In addition, Milgrom and Weber (1982) show for single-item auctions that if there is affiliation in the values of bidders, then sealed-bid auctions are less efficient than iterative auctions.

For situations with multiple items but unit demand (Demange et al. 1986) and for multiple homogeneous goods with marginal decreasing values (Green and Laffont 1979, Holmstrom 1979), it has been shown that there are generalizations which can be used to implement efficient, strategy-proof mechanisms. Finding efficient auctions with strong incentive properties turns out to be much harder for CAs with general valuations. While strategy-proofness might not be possible, researchers have been trying to find iterative CAs which still satisfy a strong solution concept, such as an ex-post equilibrium, or at least a Bayes-Nash equilibrium. For the design of electronic multi-item markets it is of significant interest whether such auction designs exist at all, and which assumptions they require. Ex post equilibria in particular avoid speculation about other bidders' valuations and could therefore reduce the strategic complexity for bidders considerably, leading to higher efficiency, and also an increased adoption of CAs.

1.1. Iterative Combinatorial Auctions

So far, the Ascending Proxy auction (Ausubel and Milgrom 2006a), iBundle(3) (Parkes and Ungar 2000), and the dVSV auction (de Vries et al. 2007) are the only known iterative combinatorial auctions which achieve full efficiency for restricted types of bidder valuations. If the coalitional value function satisfies the buyer submodularity condition[1], straightforward bidding is a best-response strategy which leads to an ex-post equilibrium and the auction results in the VCG outcome (Ausubel and Milgrom 2002). Straightforward bidding means that bidders only bid on those packages that maximize their payoff based on current ask prices in each round. These auction formats are based on non-linear and personalized prices[2] and can be modeled as an algorithm (primal-dual or subgradient) to solve the corresponding linear program. We refer to these auction formats as non-linear personalized price auctions (NLPPAs) in the following.

If the bidders' valuations in an NLPPA are not buyer submodular, bidders have an incentive to shade their bids and not follow the straightforward strategy. Buyer submodularity is equivalent to the condition that the VCG outcome lies in the core for any set of bidders, which is often not given for realistic value models (Bichler et al. 2009). Even if bidders knew that their valuations are buyer submodular and they would not need to speculate about other bidders' types, it is not obvious

[1] Bidders are more valuable when added to smaller coalitions concerning the coalitional value function

[2] The price of a package does not equal the dot product of item prices and may be discriminative for different bidders

that other bidders are able to follow the straightforward strategy in such an environment. Both computational and lab experiments have illustrated the large number of auction rounds necessary for these NLPPAs (Schneider et al. 2010), in which nearly all valuations have to be elicited to achieve efficiency. Nevertheless, these auction designs have significant theoretical value as they show restrictions under which full efficiency is possible with an ex-post equilibrium.

As an alternative, linear-price CAs have been suggested resembling the fictitious Walrasian tâtonnement. Linear prices are desirable for their simplicity and the reduced communication complexity in real world applications. One line of research is based on a restricted dual of the relaxed winner determination problem, in which the pseudo-dual variables are used as ask prices in the auction (Rassenti et al. 1982, Kwasnica et al. 2005, Bichler et al. 2009). Fluctuations of the ask prices and the complexity of the ask price calculation are problems of this approach for some applications. In contrast, Porter et al. (2003) suggest a simple mechanism with ascending linear ask prices, called the Combinatorial Clock (CC) auction.

The mechanism has achieved high levels of efficiency in the lab (Porter et al. 2003, Kagel et al. 2009, Scheffel et al. 2010) and has a number of obvious advantages. It maintains strictly ascending, linear ask prices, and limits the computational burden on the auctioneer as he only has to solve the NP-hard winner determination problem in the last rounds if there is excess supply. Also, the information revelation between rounds makes it quite robust against collusion and limits the bidder's possibilities for signaling. For these reasons, the Netherlands and the UK have recently started to use a version of the CC auction for price discovery in the sale of spectrum licenses (Cramton 2009). It is also being used in electricity markets and other high-stakes auctions, in which anonymous linear prices are often an important requirement (Cramton et al. 2006a). Unfortunately, no equilibrium strategy is known, and it is unclear for bidders which strategy they should follow.

Apart from a few lab experiments, little theoretical research has focused on the CC auction as of yet. Ausubel et al. (2006) argue that anonymous linear prices are not generally rich enough to yield efficient outcomes. The arguments are based on Ausubel and Milgrom (2002), who show that with linear prices bidders have an incentive to engage in demand reduction to favorably impact prices, which implies that the auction outcome is not fully efficient. Therefore, the version of the CC auction used for spectrum auctions in Europe and the Clock-Proxy design extend the clock auction by an additional phase, in which sealed bids can be submitted (Cramton 2009, Ausubel et al. 2006) and a payment rule is defined with the intention of providing incentives for truthful bidding. So far, no formal equilibrium analysis for such two-phased auctions has been available and the theoretical efficiency results only consider the auction format in the second phase, where the bids are typically restricted by an activity rule and the bids submitted in the clock phase.

1.2. Contributions and Composition of this Paper

Ausubel et al. (2006) write that "in environments with complementary goods, a clock auction with a separate price quoted for each individual item cannot by itself generally avoid inefficiency." We show that an extended version of the CC auction with an appropriate price update and a Vickrey payment rule can achieve full efficiency with an ex-post equilibrium. Our theoretical analysis sheds light on the reasons for inefficiency in the CC auction and shows that, in contrast to the widespread belief, also linear-price combinatorial auctions can lead to full efficiency. The work relates to an increasing number of IS articles using game theory to analyze electronic markets (Hu et al. 2004, Bapna et al. 2010, Liu et al. 2010, Greenwald et al. 2010).

Mathematical models of auctions and markets have also been criticized as unrealistic, as some of the assumptions are too strong and do not hold in practical applications (Rothkopf and Harstad 1994). For example, the celebrated Arrow-Debreu model assumes continuous, monotonic, and strictly concave utility functions and was heavily criticized for being unrealistic (Georgescu-Roegen 1979). Existing game-theoretical models of iterative combinatorial auction formats such as the

Ascending Proxy Auction (Ausubel and Milgrom 2006a), iBundle(3) (Parkes and Ungar 2000) or dVSV (de Vries et al. 2007) require the buyer submodularity condition to hold in order to achieve an ex-post equilibrium strategy. Often this condition does not hold, and even for small sized CAs, these auction formats lead to a prohibitive number of auction rounds (Schneider et al. 2010). Nevertheless, the models are significant contributions to the literature, not necessarily for their immediate practical applicability with human bidders, but because they show under which conditions full efficiency with a strong solution concept can be achieved in an iterative combinatorial auction, and that this is possible at all.

In the extension of the CC auction, the CC+ auction, the number of rounds stays low, but bidders need to submit bids on all packages with a positive valuation in each round to follow the equilibrium strategy. This is only possible for small auctions with a few items. In summary, even though there are strong solution concepts and incentives to follow the equilibrium strategy, we can not assume that bidders are able to submit enough bids or follow enough auction rounds, such that the auctioneer can always find an efficient solution. We need to aim for satisficing solutions, i.e., auction designs which provide high levels of efficiency even if bidders are restricted in the number of bids that they can reasonably submit. Game-theoretical models of CAs describe situations in which the auction is efficient. They do not provide an understanding of their efficiency when bidders deviate from their equilibrium bidding strategy.

Lab experiments can provide insights into how bidders behave in CAs, but they are costly and limited to a small number of experiments. We argue that in addition to game-theoretical modeling, computational experiments provide an important complement for understanding the robustness of theoretical results against strong assumptions, which are often required in theory. Such a sensitivity analysis is important for applications in the field and respective systems, but often beyond what can be achieved with formal models. Computational experiments should not be used instead of, but in addition to formal models. Although computational experiments are common in the CS and OR literature, they are rarely used in Microeconomics.

We also describe the results of computational experiments with different value models from CATS (Leyton-Brown et al. 2000) and analyze the impact of bidding strategies, which we observed in the lab in a large number of computational experiments. For example, we look at bidders who are limited in the number of bids they can provide in each round, or such who randomly select some packages from those with the highest payoff. We show that in smaller value models with up to nine items, the CC and the CC+ auctions achieve high levels of efficiency even if bidders are restricted in such ways, while the efficiency decreases with a larger number of items in the auction. This explains the high efficiency results observed in lab experiments (Porter et al. 2003, Scheffel et al. 2010, Kagel et al. 2009), but it also highlights that such results do not necessarily carry over to auctions with more than ten items.

In Section 2 we summarize related literature on linear competitive equilibrium prices. In Section 3 we present analytical results on the worst case efficiency of the CC auction assuming simple bidding strategies. We assume straightforward bidding, i.e., truthful revelation of the payoff-maximizing packages in response to ask prices (Parkes 2006), since this strategy is easy to follow and limits the amount of information that needs to be revealed in each round. We introduce a *demand-masking set of valuations* and show that the efficiency of the CC auction can be as low as 0%. While the example that leads to 0% efficiency can be considered a degenerate case, we also discuss situations that we found regularly in numerical experiments with realistic value models, and which can also lead to efficiencies as low as 50% with straightforward bidding. As an alternative, we evaluate a powerset strategy in which bidders bid on all possible packages with positive payoff in each round. We show, however, that even if bidders reveal as much information in each round, the efficiency of the CC auction can also decrease to 0%. This analysis helps to understand situations in which the CC auction is inefficient and to propose improvements.

Based on these results, in Section 4 we identify properties of an auction mechanism that satisfy efficiency with a strong game-theoretical solution concept. We suggest a variation of the CC auction, the CC+ auction, which leads to 100% efficiency with powerset bidding. First, we modify the price update rule to allow full efficiency. Second, we introduce the VCG payment rule to ensure incentives for truthful bidding. Then, we show that powerset bidding becomes an ex-post equilibrium strategy for general valuations in the CC+ auction. This result might seem surprising due to the negative results on the efficiency of linear competitive equilibrium prices in the literature (Gul and Stacchetti 1999). It is, however, possible due to the distinction between final ask prices and payments in the CC auction.

Some assumptions of the CC+ auction are also strong and might not be given in practical applications. In particular, a powerset equilibrium bidding strategy is only viable for smaller instances with a few items. In Section 5 we provide results of computational experiments of iBundle, the CC, and the CC+ auction and restrict bidders in the number of package bids they can submit in each round. We show that all auction formats achieve high levels of efficiency beyond 90% in smaller value models. It is interesting to focus on the comparison of the CC and the CC+ auction and those strategies in which bidders are heavily restricted in the number of bids (up to 10) they can submit in each round. For smaller value models with bidders interested in up to 129 packages, the CC+ yields a significantly higher efficiency than the CC auction, beyond 98%. In larger value models with bidders interested in 443 or 32,767 packages, this advantage vanishes and there is no longer a significant difference between the efficiency of the CC and the CC+ auction. Still, the average efficiency in this Real Estate 5x3 model is beyond 92% with restricted bidders. Note that even in large value models, we do not expect human bidders to assign a positive valuation for several hundred packages. Even in spectrum auctions with hundreds of adjacent regions, one cannot expect a bidding team to assign reasonable values for several hundred or thousands of packages. Note also that previous research shows that combinatorial auctions still achieve considerably higher efficiency than non-combinatorial auction formats in the presence of complementarities among items (Banks et al. 2003).

The simulations also highlight some virtues of the CC and the CC+ auction compared to non-linear and personalized price auctions such as iBundle. The number of auction rounds of the CC and the CC+ auction are similar, but much lower than those of the iBundle auction. The number of bids submitted in iBundle is orders of magnitude higher than in the CC+ auction, although the efficiency is not worse. This might well make a difference in practical applications.

2. Related Theory and Definitions

Achieving efficiency on markets when economic agents strategically pursue their individual self-interest is a fundamental problem in Economics. General equilibrium models showed that in classical convex economies with multiple products, the Walrasian price mechanism verifies the efficiency of a proposed allocation (Arrow and Debreu 1954) while communicating as few real variables as possible (see Mount and Reiter (1974) and Hurwicz (1977)). Furthermore, Jordan (1982) shows that the Walrasian mechanism is a unique voluntary mechanism with this property. However, these results assume that all production sets and preferences are convex and do not apply to non-convex economies with indivisible goods, such as combinatorial auctions. As already discussed, such neoclassical general equilibrium models have often been criticized for their strong assumptions (Georgescu-Roegen 1979).

Bikhchandani and Mamer (1997) show that without convexity assumptions full efficiency cannot be achieved with linear *competitive equilibrium* (CE) prices for general valuations (see Nisan and Segal (2006) for an overview). Later, Gul and Stacchetti (1999) proved that for all bidders it is almost necessary that *goods are substitutes* to ensure efficiency with linear CE prices. So far, only *iterative combinatorial auction* (ICAs) designs with non-linear and personalized prices have been shown to be fully efficient.

In most practical applications of ICAs, linear and anonymous ask prices are essential. For example, day-ahead markets for electricity sacrifice efficiency for the sake of having linear prices (Meeus et al. 2009). Also, the main auction formats which have been used or discussed for selling spectrums in the US use linear prices (Brunner et al. 2009). The CC auction is probably the most widespread ICA format, but the negative results by Gul and Stacchetti (1999) seem to indicate that there is no hope of making the CC auction fully efficient for general valuations.

A notable difference between the CC auction and auctions with pseudo-dual linear prices, for example, is that bidders need not pay the ask prices of the final round. The winner determination in the final round can select a bid and the corresponding ask price from a previous round, so that there is a distinction between ask prices and payments. This distinction opens up the possibility of achieving efficiency with linear ask prices and a strong game-theoretical solution concept for general valuations in the CC auction. The latter is important, as any restriction on the valuations is typically unknown.

We introduce the necessary notation and review relevant theory on linear-price CAs. There is a set $\mathcal{K}$ of m indivisible items indexed with k or l, which are auctioned among n bidders. Let $i, j \in \mathcal{I}$ denote the bidders and $v_i : S \to \mathbb{R}$ denote a value function of bidder i, which assigns a real value to every subset $S \subseteq \mathcal{K}$ of items. An allocation $X \in \Gamma$ of the m items among bidders is $X = \{X_1, ..., X_n\}$, with $X_i \cap X_j = 0$ for every $i \neq j$. X_i is the package of items assigned to bidder i. The social welfare of an allocation X is $\sum_{i \in \mathcal{I}} v_i(X_i)$, and an efficient allocation X^* maximizes social welfare among all allocations X, such that $\forall X, \sum_{i \in \mathcal{I}} v_i(X_i^*) \geq \sum_{i \in \mathcal{I}} v_i(X_i)$.

We focus on linear-price CAs, in which an ask price β_k for each of the m items is available; the price of a package S is the sum of the prices of the items in this package. We assume that the *demand* of each bidder are the packages which maximize his utility.

Definition 1. (Blumrosen and Nisan 2007) For a given bidder valuation v_i and given item prices $\beta_1, ..., \beta_m$, a package $R \subseteq \mathcal{K}$ is called a *demand* of bidder i if for every other package $S \subseteq \mathcal{K}$ we have that $v_i(S) - \sum_{k \in S} \beta_k \leq v_i(R) - \sum_{k \in R} \beta_k$.

A feasible allocation X and a price vector β_k are in competitive equilibrium (CE) when the allocation maximizes the payoff of every bidder and the auctioneer given the prices. A *Walrasian equilibrium* can then be described as a vector of item prices.

Definition 2. A Walrasian equilibrium is a set of nonnegative prices $\beta_1, ..., \beta_m$ and an allocation X if for every player i, X_i is the demand of bidder i at those prices and for any item k that is not allocated $\beta_k = 0$.

Simple examples illustrate that Walrasian equilibria do not exist for general valuations in CAs if goods are indivisible; in other words, for certain types of bidder valuations it is impossible to find linear CE prices which support the efficient allocation X^* (Blumrosen and Nisan 2007). Let us assume that bidder 1 has a value of 10 for the items (1), (2), and also for the package (1, 2), and bidder 2 has only a positive valuation of 12 for the package, but not for the singletons. The optimal allocation is to allocate the package (1, 2) to bidder 2. The prices will be 10 for each item, otherwise bidder 1 would demand one of the items, and consequently 20 for the package (1, 2). Bidder 2 will, however, not demand the package at a price of 20, and no equilibrium exists.

The economic "goods are substitutes" property is a sufficient condition for the existence of Walrasian equilibrium prices (Kelso and Crawford 1982). Intuitively, this property implies that every bidder continues to demand the items which do not change in price, even if the prices on other items increase. Overall, the "goods are substitutes" condition is very restrictive as most known practical applications of CAs deal more with complementary goods.

Actually, Gul and Stacchetti (2000) show that even if bidders' valuation functions satisfy the "goods are substitutes" condition, no ascending CA exists that uses anonymous linear prices and arrives at the VCG solution. This means that bidders may have an incentive to demand smaller packages of items in order to lower their payments.

Bikhchandani and Ostroy (2002) prove that only with personalized non-linear prices does a CA always achieves a CE. The Ascending Proxy Auction, iBundle(3) and the dVSV auction are designs using non-linear personalized prices at the expense of an exponential (in m) number of auction rounds. Also, the final ask prices generated are not VCG prices for general valuations and thus bidders still might be incentivized to defect from the assumed straightforward bidding strategy. Straightforward bidding is only an ex-post equilibrium in these NLPPAs if bidder valuations are submodular (de Vries et al. 2007).

Definition 3. *Final ask prices* are the ask prices β_k of the last round of an iterative auction.

Definition 4. A *payment* is the amount of money a bidder has to pay for his winning items.

In the efficient ICAs (Ascending Proxy Auction, iBundle, dVSV), there is no difference between final ask prices and payments. They charge bidders the final ask prices to pay for their winning packages. This means ask prices need to be non-linear and personalized to guarantee efficiency. In contrast to NLPPAs, the CC auction distinguishes between final ask prices and payments. This opens up the possibility of maintaining linear ask prices, but achieving efficient solutions with a strong solution concept by implementing non-linear payments.

3. The CC Auction

We concentrate on the CC auction as introduced by Porter et al. (2003) and give a precise description in Algorithm 1. Prices for all items are initially zero. In every round bidders identify a package of items, or several packages, which they offer to buy at current prices. If two or more bidders demand an item then its price is increased by a fixed bid increment in the next round. This process iterates. The bids which correspond to the current ask prices are called *standing*, and a bidder is standing if he has at least one standing bid. In a simple scenario in which supply equals demand, the auction terminates and the items are allocated according to the standing bids. If at some point there is excess supply for at least one item and no item is over-demanded, the auctioneer determines the winners to find an allocation of items that maximizes his revenue by considering all submitted bids. If the solution displaces a standing bidder, the prices of items in the corresponding standing bids rise by the bid increment and the auction continues. The auction ends when no prices are increased and bidders finally pay their bid prices for winning packages. We analyze a version that uses an XOR bidding language.

3.1. Efficiency of the CC Auction

We analyze the worst-case efficiency of the CC auction with bidders following the straightforward strategy, which is typically assumed in game-theoretical models of ICAs. We also evaluate a powerset strategy, which describes the situation in which bidders reveal all packages with a positive valuation at the current prices. We draw on this strategy in subsequent sections.

Definition 5. A *straightforward* bidder bids only for his *demand* in each round at the current ask prices $\beta_1, ..., \beta_m$.

Note that a straightforward bidder might bid on several packages in a round if they apply to the definition of demand (cf. Definition 1).

Definition 6. The *powerset* bidder bids on all packages S with a non-negative value $v_i(S) - \sum_{k\in S}\beta_k \geq 0$ at the current set of ask prices $\beta_1, ..., \beta_m$.

We show that if all bidders follow the straightforward strategy, the efficiency of the CC auction can be as low as 0%. For this, we refer to a recent theorem by Kagel et al. (2009) on the efficiency of auctions which maximize the auctioneer's revenue based on bid prices.

A *standard* package auction is defined such that it selects an allocation $\overline{X}$ to maximize the auctioneer's revenue $\overline{X} \in \arg\max_X \sum_{i\in\mathcal{I}} \beta_i(X_i)$ and has bidder i pay $\beta_i(\overline{X}_i)$. $\beta_i(X_i)$ denotes the highest price that i bids for a package X_i during the course of the auction.

```
Data: package bids β_i(S)
Result: allocation X̄ and prices β_i(X̄_i)
initialization
  for k=1 to m do β_k ← 0
  for i=1 to n do X_i ← ∅
repeat
  overdemand ← FALSE; undersupply ← FALSE
  for i=1 to n do
    bidders submit bids β_i(S)
  for k=1 to m do
    if ≥ 2 bidders i ≠ j demand item k then
      β_k ← β_k + ϵ
      overdemand ← TRUE
    end
    if item k is not part of a bid β_i(S) then
      undersupply ← TRUE
    end
  if overdemand = TRUE then exit iteration
  else if undersupply = FALSE then exit loop
  else
    for k=1 to m do
      Assign β_i(S) with k ∈ S to the set of standing bids B
      Calculate X̄ based on all bids submitted in the auction
        if a bidder holding a bid in B is displaced, i.e. no bid by this bidder is in X̄, then
          foreach item k which was displaced: do
          β_k ← β_k + ϵ
          end
        else X̄ is the final allocation
    end
until stop
```

Algorithm 1: CC auction

A standard package auction can be modeled as a cooperative game with transferable utility, in which the payoff vector or imputation π is given by the auctioneer's revenue $\pi_0 = \sum_{i\in\mathcal{I}} \beta_i(\overline{X}_i)$, and bidder i's payoff $\pi_i = v_i(X_i) - \beta_i(X_i)$. The value of a coalition including the auctioneer and the bidders in $T \subseteq \mathcal{I}$ is $w(T) = \sum_{i\in T} v_i(X_i^*|_T)$.

A feasible allocation X with prices β and a corresponding imputation π is a *core* allocation if, for every set of bidders $T \subseteq \mathcal{I}$, the imputation satisfies $\pi_0 + \sum_{i\in T}\pi_i \geq w(T)$. A set of bidders T is *relevant* if there is some imputation such that $\pi_0 + \sum_{i\in T}\pi_i = w(T)$. The package X_i is the respective efficiency-relevant package.

Theorem 1. *(Kagel et al. 2009) In a standard package auction, if for some relevant allocation $X \in \arg\max_X \sum_{i\in\mathcal{I}} v_i(X_i)$ and for all bidders i, $v_i(X_i) - \beta_i(X_i) \leq \overline{\pi}_i$, then the allocation $\overline{X}$ is efficient: $\overline{\pi}_0 + \sum_{i\in\mathcal{I}}\overline{\pi}_i \geq w(\mathcal{I})$. If the efficient allocation is unique, then the auction outcome $\overline{X}$ is efficient only if for every bidder i, $v_i(X_i) - \beta_i(X_i) \leq \overline{\pi}_i$.*

To promote these results, the auction mechanism must encourage bidders to bid aggressively all the way up to their full values ($\beta_i(X_i) = v_i(X_i)$) for *efficiency-relevant packages*, i.e., packages that may become winning packages.

	$\beta_{(1)}$	$\beta_{(2)}$	$\beta_{(3)}$	...	(1)	(2)	(3)	...	(1,2)	(1,3)	...
v_1					10*						
v_{2_a}						4*			10		
v_{2_b}									10		
v_{3_a}							4*			10	
v_{3_b}										10	
...								...			...
$t=1$	1	1	1	...	1_1				$2_{2_a,2_b}$	$2_{3_a,3_b}$	...
$t=2$	2	2	2	...	2_1				$4_{2_a,2_b}$	$4_{3_a,3_b}$	...
$t=3$	3	3	3	...	3_1				$6_{2_a,2_b}$	$6_{3_a,3_b}$	...
$t=4$	4	4	4	...	4_1				$8_{2_a,2_b}$	$8_{3_a,3_b}$	...
$t=5$	5	5	5	...	5_1				$10_{2_a,2_b}$	$10_{3_a,3_b}$	...
$t=6$	6	6	6	...	6_1						
$t=7$	7	6	6	...	7_1						
...											
$t=10$	10	6	6	...	10_1						

Table 1 **Example of a demand masking set of bidder valuations and CC auction process assuming straightforward bidders.**

3.2. Worst-case Efficiency of the CC Auction with Straightforward Bidders

If a bidder follows the straightforward strategy in the CC auction, he does not bid on all relevant packages in the course of the auction. The example in Table 1 illustrates a characteristic situation that we refer to as *demand masking set.* The upper part of the table describes valuations of $2m-1$ bidders for m items, while the lower part shows both ask prices for items and bid prices for packages in individual rounds t. The indices of the bid prices for different packages indicate which straightforward bidder submits the bid on the respective package. There is one bidder called bidder 1 and for each $h \in \{2,...,m\}$ there are two bidders h_a and h_b. Bidder 1 values item (1) at a value of 10 and does not value any other item. For $h = 2,...,m$, bidders h_a and h_b value the package (1,h) at 10 and bidders h_a the item (h) at 4, and are not interested in any other package. Without loss of generality, we assume a bid increment of 1. Straightforward bidders h_a and h_b demand the package (1,h) until round 6, at which point they demand nothing. After round 6 there is excess supply and the auctioneer solves the winner determination problem, which displaces the sole remaining standing bidder, who bids on item (1). Thus the price on item (1) further increases until bidder 1 wins item (1) in round 10, and the auction terminates with a social surplus of 10. However, the efficient allocation assigns item (1) to bidder 1, and item (h) to bidder h_a for a social welfare of $10+4(m-1)$. $10/(10+4(m-1))$ converges to 0 as m approaches infinity.

We provide a formal definition of a demand masking set and derive a worst-case bound for these situations as a function of m.

Definition 7. A *demand masking* set of bidder valuations is given if the following properties are fulfilled. There is a set of bidders $\mathcal{I}$ with $|\mathcal{I}| \geq 3$, a set of items $\mathcal{K} = \{1,...,m\}$ with $R \subseteq \mathcal{K}$ and a partition $\mathcal{H}$ of $\mathcal{K} \backslash R$. Let S_h be the elements of $\mathcal{H}$ with $h \in \{2,...,|\mathcal{H}|+1 = g\}$. For each S_h there are two bidders h_a and h_b. Bidder 1 values package R with ξ. For $h \in \{2,...,g\}$ bidders h_a value the packages S_h with ν_h and $R \dot\cup S_h$ with μ and bidders h_b value only package $R \dot\cup S_h$ with μ. No bidders are interested in the other packages, i.e., the marginal value of winning any additional item to the positively valued packages is zero.

Note that the valuations of zero as shown in Table 2 do not need to be strictly zero, but rather sufficiently small so as not to influence the economy.

Theorem 2. *If bidder valuations are demand masking and all bidders follow the straightforward strategy in the CC auction, then the efficiency converges to* $\frac{2}{m+1}$ *in the worst case.*

	R	$\{S_h\}$	$\{R\dot{\cup}S_h\}$
v_1	ξ	0	ξ
$\{v_{h_a}\}$	0	ν_h	μ
$\{v_{h_b}\}$	0	0	μ

Table 2 Demand masking set of bidder valuations.

The proof is provided in Appendix A.

In the example of Table 1, ν_h is smaller than 5 for all h. With $m = 3$ and $\nu_h = \nu = 5 - \rho$ for all h, efficiency is approximately $50\% = 10/(10 + \nu(m-1))$, which is equal to $2/(m+1)$ in the worst case. Obviously if the number of items m and the corresponding number of bidders increases to fulfill the requirements of a demand masking set, efficiency converges to 0% in the worst case. While such a situation that leads to 0% efficiency can be considered a degenerated case that does not happen that often in practice, we found regular situations in simulations with realistic value models in which the case of $m = 2$ or $m = 3$ occurred, which still leads to efficiencies of 67% or 50% in the worst case. Note that these are not necessarily the only characterizations of value models in which such low efficiency can occur.

3.3. Worst-Case Efficiency of the CC Auction with Powerset Bidders

One of the reasons for the popularity of ascending auctions is that they require only partial revelation of the private information (Blumrosen and Nisan 2007). In a CA this is less of an advantage, as it is still necessary to elicit all valuations except those of the winning bids in the efficient allocation in the worst case. This means that if there are z winning package bids in an efficient allocation, $n2^m - z$ valuations need to be elicited by the auctioneer to guarantee full efficiency. For example, ascending auctions with non-linear personalized prices such as iBundle (Parkes and Ungar 2000), the Ascending Proxy Auction (Ausubel and Milgrom 2002), or dVSV (de Vries et al. 2007) are protocols that in each round elicit the demand set of each bidder and provably find an efficient solution at the expense of an exponential number (in m) of auction rounds (Blumrosen and Nisan 2007). In such an NLPPA with straightforward bidders at least all valuations of all losing bidders are elicited.

As an alternative to straightforward bidding, the auctioneer can try to encourage bidders to bid on many packages from the start. In the best case, bidders reveal all packages with positive payoff, i.e., they follow a powerset strategy. Unfortunately, even if bidders follow the powerset strategy, the CC auction does not necessarily terminate with an efficient solution.

PROPOSITION 1. *If all bidders follow the powerset strategy, the efficiency of the CC auction converges to 0% in the worst case.*

The proof and an example are provided in Appendix A.

Inefficiencies in the CC auction with powerset bidders occur if there are two overlapping packages by the winning bidder, and there is only competition on the package with the lower valuation. This drives up the prices only on the lower valued package, which is finally sold, although the bidder has a higher valuation for the other package, for which he cannot increase his bid.

3.4. Modifications of the CC Auction

The analysis in Section 3.3 shows that even if bidders reveal all profitable packages in each round, the CC auction can be inefficient. However, a small change in the price update rule allows all losing package valuations to be elicited and makes the CC auction fully efficient with powerset bidders.

DEFINITION 8. A *partial revelation price update rule* in the CC auction also increases prices for each overdemanded item and in addition for each item of a standing bid which is displaced by the winner determination.

The difference to the original price update rule is very small. While the original CC auction terminates if all bidders holding a standing bid get any package in the final allocation (not necessarily one of their standing bids), the partial revelation price update rule requires a bidder to get exactly his standing bid allocated. Thus one bidder holding two or more standing bids causes prices to increase and the auction to continue.

Corollary 1. *If all bidders follow the powerset strategy, the CC auction with the partial revelation price update rule terminates with an efficient outcome.*

The proof is provided in Appendix A.

The auction can still suffer from small inefficiencies due to the minimal bid increment. *Last-and-final bids* have been suggested as a means to get rid of these inefficiencies (Parkes 2006). They allow bidders to submit a final bid on a package which is above the ask price of the previous round, but below the current ask price for a package. For the sake of clarity, we omit this rule in our analysis.

4. The CC+ Auction

Even if the powerset strategy leads to full efficiency in a modified CC auction with linear ask prices, it is not obvious why a bidder should follow the powerset strategy. We show that the powerset strategy is an ex-post equilibrium, but that it requires an even stronger price-update rule and a VCG payment rule (Ausubel and Milgrom 2006b). We refer to this auction design as a CC+ auction. A description of the CC+ auction with powerset bidders is provided in Algorithm 2. Modifications to the original CC auction are underlined.

Definition 9. A *full revelation price update rule* in the CC+ auction increases prices on items as long as at least a single bidder bids on the item.

We aim for a strong game-theoretical solution concept. A desirable property is a profile of strategies with an ex-post equilibrium, in which a bidder does not regret his bid even when he is told what everyone's type is after the auction. Note that we are not attempting to achieve a dominant strategy equilibrium, as preference elicitation in an indirect mechanism can invalidate dominant strategy equilibria existing in a single-step version of a mechanism (Conitzer and Sandholm 2002). We discuss the types of speculation that are possible in a CC+ auction with full information in Appendix B. It illustrates that ex-post equilibria are not as strong as dominant strategy equilibria, but they are much stronger than Bayesian Nash equilibria, because they do not require agents to speculate on other bidders' types or valuations. When iterative preference elicitation is used to implement a mechanism which is a dominant-strategy direct-revelation mechanism in a sealed-bid version, then each agent's best (even in hindsight) strategy is to act truthfully if the other agents act truthfully (Conen and Sandholm 2001).

Definition 10. Truthful bidding in every round of an auction is an *ex-post equilibrium* if for every bidder $i \in \mathcal{I}$; if bidders in $\mathcal{I}_{-i}$ follow the truthful bidding strategy, then bidder i maximizes his payoff in the auction by following the truthful bidding strategy (Mishra and Parkes 2007).

4.1. Efficiency and Incentive Compatibility of the CC+ Auction

We show that the CC+ auction maintains linear ask prices and achieves an efficient solution, while being incentive compatible. Note that we do not need to make any restrictive assumptions on the bidders' valuations. To prove the efficiency already, the slightly weaker partial revelation price update rule is sufficient (cf. proof for Corollary 1).

Corollary 2. *A powerset strategy is an ex-post equilibrium in the CC+ auction.*

The proof is provided in Appendix A.

As all bidders reveal all valuations, a bidder cannot improve his payoff by unilaterally deviating from the truthful powerset strategy in a respective CC+ auction, or influence whether the other

```
Data: package bids β_i(S)
Result: efficient allocation X* and prices β_i(X_i*)
initialization
  for k=1 to m do β_k ← 0
  for i=1 to n do X_i ← ∅
repeat
  overdemand ← FALSE; undersupply ← FALSE
  for i=1 to n do
    submit a bid β_i(S) on each package S, which applies to v_i(S) − Σ_{k∈S}(β_k) ≥ 0
  for k=1 to m do
    if ≥ 1 bidders demand item k then
      β_k ← β_k + ε
      overdemand ← TRUE
    end
    if item k is not part of a bid β_i(S) then
      undersupply ← TRUE
    end
  if overdemand = TRUE then exit iteration
  else if undersupply = FALSE then exit loop
  else
  Calculate the final allocation X* based on all submitted bids
  exit loop
until true
Calculate VCG prices β*_VCG based on all submitted bids
```

Algorithm 2: CC+ auction with powerset bidding

bidders reveal their valuations truthfully. Therefore, the bidder's truthful powerset strategy is independent of the other bidders' types. This result shows what types of price update and payment rules are sufficient for a powerset strategy to satisfy an ex-post equilibrium.

While the partial revelation price update rule is sufficient for efficiency, when all bidders follow a powerset strategy, a full revelation price update rule is necessary to achieve an ex-post equilibrium.

PROPOSITION 2. *Powerset bidding does not satisfy an ex-post equilibrium in the CC+ auction with only a partial revelation price update rule.*

The proof is provided in Appendix A.

4.2. Communication Complexity of the CC+ Auction

Nisan and Segal (2006) show that determining an optimal allocation requires an exponential number of queries from the auctioneer to the bidders. There are subtle differences, however, in the amount of information that is elicited by different auction formats. A VCG auction and a CC+ auction ask bidders to reveal all $n2^m$ valuations to the full extent. In a CC+ auction, a bidder sees the price clock increase on various items and learns at which prices nobody demands a particular item any more. In a VCG auction, bidders only know that a bid on a particular package was lost. In both cases, the auctioneer learns all valuations of all bidders. Using the partial revelation price update rule in the CC+ auction with z winning bids, only $n2^m - z$ losing valuations are elicited.

In NLPPAs such as the Ascending Proxy Auction, iBundle(3), or dVSV, the auctioneer elicits $n2^m - z$ preferences in the worst case. It might also be that the winners do not need to reveal all valuations on losing packages. However, a strong solution concept is only satisfied if buyer submodularity is given. Clearly, communication complexity will always remain a stumbling block for any of the theoretical models in situations with more than a few items only. The assumption of following a straightforward strategy in exponentially many auction rounds only holds in automated settings with proxy agents. The same is true for the powerset strategy, even if the number of auction rounds is much lower. We address this issue and the robustness of the efficiency results with respect to deviations from the powerset strategy in Section 5.

Similar to work on NLPPAs, the CC+ auction is, however, of theoretical value as it shows sufficient rules and assumptions to design an ascending CA that uses linear ask prices and achieves an efficient outcome with a strong solution concept for general valuations. This provides a theoretical foundation for combinatorial clock auctions.

4.3. Alternative Payment Rules in the CC+ Auction

The CC+ auction suffers from some of the problems of the VCG design, in particular that the outcome might not be in the core (Ausubel and Milgrom 2006b). In other words, there are some bidders who could make a counteroffer to the auctioneer that both sides would prefer to the VCG outcome. In such situations, the auctioneer can increase his sales revenue by excluding certain bidders, which is also referred to as revenue non-monotonicity. The bidders could also increase their payoff through shill bidding. These vulnerabilities of VCG outcomes are considered serious problems for applications in the field. In some settings, it is sufficient to have a mechanism which is in the core, but which is as close to incentive compatibility as possible.

Day and Raghavan (2007) have recently suggested bidder-Pareto-optimal prices in the core as an alternative to VCG prices. An outcome of an auction is bidder-Pareto-optimal in the core if no Pareto improvement is possible within the core. This means that if we lower one bidder's payment, some other bidder's payment must increase to remain in the core. Such an outcome minimizes the total payments within the core.

Definition 11. (Day and Raghavan 2007) An outcome is *bidder-Pareto-optimal* if there is no other core outcome weakly preferred by every bidder and strictly preferred by at least one bidder in the winning coalition.

Note that if items are complements, core prices exceed VCG prices strictly. Day and Milgrom (2007) show that a core-selecting auction provides minimal incentives for bidders to deviate from truthful reporting, if it chooses a bidder-Pareto-optimal outcome. Day and Raghavan (2007) also describe a constraint generation approach that generates bidder-Pareto-optimal core prices rapidly for sealed bid auctions. The payment scheme minimizes the total availability of gains from unilateral strategic manipulation. The final bids of each bidder on all packages in a CC+ auction can also be used to calculate bidder-Pareto-optimal core prices.

Corollary 3. *The CC+ auction with powerset bidders terminates with a core outcome if it charges bidder-Pareto-optimal prices as payments instead of VCG prices.*

The proof is provided in Appendix A.

Note that even with the weaker partial revelation price update rule, Corollary 3 holds. In contrast to the Clock-Proxy auction (Ausubel et al. (2006)), bidders in the CC+ auction do not need to type in valuations to a proxy agent after the CC auction has finished, and the bidder-Pareto-optimal prices are calculated right away.

5. Computational Experiments

In the previous section, we show that the powerset strategy leads to efficiency in the CC+ auction. Powerset bidding is typically not viable for bidders except for small CAs. So far, only a few papers provide results on individual bidding behavior in CAs. Scheffel et al. (2010) report that lab subjects submit around 10 to 12 bids per round in linear-price auctions independent of the number of packages with a positive valuation. Kagel et al. (2009) report that bidders bid only on a fraction of the profitable packages in the CC auction. Global bidders bid between 12 snd 14 percent of the profitable packages in one treatment with six items and 21 to 28 percent in a treatment with four items.

This section describes the results of computational experiments and analyzes efficiency, revenue, number of auction rounds, and the number of submitted bids with artificial bidders in the CC and variations of the CC+ auction with respect to deviations from the powerset strategy. The bidding agents follow either the straightforward or the powerset strategy, plus we also implement agents with restrictions on the number of packages submitted in each round. We compare the results to iBundle, which uses non-linear and personalized prices. The experiments help to understand how restricted communication impacts efficiency in the CC+ auction. In contrast to the worst-case analysis that we provide in the first sections of this paper, this section provides more of an average case analysis for different bidder types, based on realistic value models.

5.1. Experimental Setup

The experimental setup is based on three treatment variables, namely the value model, the bidding strategy, and the auction format.

5.1.1. Value Models. Since there are hardly any real-world CA data sets available, we base our experiments on synthetic valuations generated with the Combinatorial Auctions Test Suite (CATS) (Leyton-Brown et al. 2000).

The ***Transportation*** value model uses the *Paths in Space* model from the CATS. It models a nearly planar transportation graph in Cartesian coordinates, in which each bidder is interested in securing a path between two randomly selected vertices (cities). The items traded are edges (routes) of the graph. Parameters for the Transportation value model are the number of items (edges) m and graph density η, which defines an average number of edges per city, and is used to calculate the number of vertices as $(2m)/\eta$. The bidder's valuation for a path is defined by the Euclidean distance between two nodes multiplied by a random number, drawn from a uniform distribution. Consequently only a limited number of packages, which represent paths between both selected cities, are valuable for the bidder. This allows the consideration of even larger transportation networks in a reasonable time. In this work we use a value model with 25 items and 15 bidders. Every bidder has interest in 16 different packages on average.

The ***Real Estate 3x3*** value model is based on the *Proximity in Space* model from the CATS. Items sold in the auction are the real estate lots k, which have valuations $v(k)$ drawn from the same normal distribution for each bidder. Adjacency relationships between two pieces of land p and q (e_{pq}) are created randomly for all bidders. Edge weights $r_{pq} \in [0,1]$ are then generated for each bidder, and they are used to determine package valuations of adjacent pieces of land:

$$v(S) = (1 + \sum_{e_{pq}:p,q \in S} r_{pq}) \sum_{k \in S} v(k)$$

In this work we use the *Real Estate 3x3* value model with nine lots for sale. Individual item valuations have a normal distribution with a mean of 10 and a variance of 2. There is a 90% probability of a vertical or horizontal edge, and an 80% probability of a diagonal edge. Edge weights have a mean of 0.5 and a variance of 0.3. All experiments with the Real Estate 3x3 value model are conducted with five bidders, who are interested in a maximum package size of 3, because large packages are always valued more highly than small ones. This is also motivated by real-world observations by An et al. (2005), in which bidders typically have an upper limit on the number of items they are interested in. Without this limitation, the auction easily degenerates into a scenario with a single winner for the package containing all items.

In order to analyze a value model with many items, a very large number of possible packages for each bidder, and the impact of the threshold problem, we also use a ***Real Estate 3x5*** value model. This model contains two different bidder types one big bidder interested in all 15 items, and five smaller bidders. Each small bidder is interested in a randomly determined preferred item, all horizontally and vertically adjacent items and the items adjacent to those. This means that a small bidder is typically interested in six to eleven items with local proximity to their preferred item. For each bidder we draw the baseline item valuation $v_i(k)$ from a uniform distribution separately. Complementarities occur upon vertical and horizontal adjacent items based on a logistic function to determine package valuations: $v_i(S) = \sum_{C \in P} \left(\left(1 + \frac{a}{100\left(1+e^{b-|C|}\right)} \right) * \sum_{k \in C} v_i(k) \right)$, with P being the partition of S containing maximal connected packages C. For our simulations we choose $a = 340$ and $b = 8$ for the big bidder and $a = 160$ and $b = 4$ for all small bidders, and draw the baseline valuations for the big bidder on the range $[3, 9]$ and for the small bidders on the range $[3, 20]$.

The size of a value model describes the number of possible bids which a bidder can evaluate. While in the Transportation value model bidders are interested in only 16 packages on average and in the Real Estate 3x3 value model in 129, small bidders in the Real Estate 3x5 value model are interested in 443 packages on average and the big bidder is interested in $2^{15} - 1 = 32,767$ packages. We find that the size of the value model has an impact on the average efficiency achieved if bidders do not reveal all their valuations throughout the auction, as is the case with a straightforward strategy in iBundle or a powerset strategy in the CC+ auction.

Since we find similar results in other models, we concentrate only on the ones described above for clarity and move the others to Appendix D.

5.1.2. Bidding Agents. In our theoretical analysis, we introduce the straightforward and the powerset strategies. The ***powerset*** bidder evaluates all possible packages in each round, and submits bids for all packages which are profitable given current prices. In addition to the powerset bidder, we analyze bidders which are restricted to bid only on the best six or the best ten packages in each round, similar to bidders in the lab. These bidders choose those packages with the highest payoff. Inspired by observations in the lab, we also model a ***heuristic 5of20*** bidder. This bidder randomly selects five out of his 20 best packages based on his payoff in a round. This bidder allows the evaluation of the robustness of the auction against randomness in the bidding strategies.

In contrast, the ***straightforward*** bidder only bids on his demand in each round, i.e., on those package(s) that maximize his payoff given current prices.

5.1.3. Treatment Structure We use a $7 \times 6 \times 5$ factorial design (cf. Table 3), in which all value models are analyzed in different auction formats with all of the above bidding strategies. Each treatment is repeated 50 times with different random seeds for value models and bidding strategies, resulting in 10,500 auctions. The auctions use a minimum increment of 1 and the XOR bidding language.

Value Model		Auction Format		Bidding Strategy
Transportation		CC		Straightforward
Real Estate 3x3		CC+ (partial, Core)		Preselect 10
Real Estate 5x3		CC+ (full, Core)		Heurisitc 5 of 20
Transportation large (Appendix D)	×	CC+ (partial, VCG)	×	Powerset6
Pairwise Synergy low (Appendix D)		CC+ (full, VCG)		Powerset10
Pairwise Synergy high (Appendix D)		iBundle		Powerset
Airports (Appendix D)				

Table 3 Treatment factors.

5.2. Experimental Results

We present the aggregate results of our computational experiments with the three different value models[3]. We evaluate straightforward and powerset bidders, but also bidders following heuristic bidding strategies in order to provide an indication of the impact of heuristic bidding strategies as they can be found with human bidders in the lab or in the field on efficiency.

The results are presented in Tables 4 to 6 and in Appendix D. We measure mean and minimum efficiency, and mean revenue to characterize the auction outcome. Furthermore, we compare number of rounds and total number of bids submitted by the bidders. iBundle leads to a very large number of auction rounds in all but small value models. For the Real Estate 5x3 value model the computation time was such that only a single auction took over 60 hours and 500 rounds as the winner determination takes increasing amounts of time. We decided not to report iBundle results on this value model, as such auctions would not be conducted with human bidders in the field.

Measure	Bidder Type	Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	99.48	97.02	97.38	96.96	96.83
	CC+ (partial)	99.52	99.87	99.86	99.85	99.92
	CC+ (full)	99.62	99.88	99.84	99.87	99.93
	iBundle	100.00	93.74	97.54	97.89	97.22
Min. Efficiency in %	CC	94.81	84.65	86.71	83.22	83.15
	CC+ (partial)	94.81	96.69	96.69	96.69	98.60
	CC+ (full)	94.81	96.69	96.69	96.69	98.60
	iBundle	100.00	74.72	85.71	89.44	74.56
Mean Rounds	CC	29.10	25.36	25.22	25.10	24.96
	CC+ (partial)	29.04	31.22	31.40	31.10	30.90
	CC+ (full)	44.78	37.80	37.94	37.50	37.26
	iBundle	77.08	277.84	193.48	130.44	75.86
Mean # of Bids	CC	295.18	452.50	479.92	562.50	805.88
	CC+ (partial)	295.12	475.72	505.56	586.30	828.92
	CC+ (full)	332.88	471.36	501.08	582.22	825.44
	iBundle	7785.48	5791.42	5051.68	4941.52	6440.32
Mean Revenue in %	CC	69.43	83.74	83.30	84.23	84.30
	CC+ (partial, Core)	55.34	58.80	58.23	58.67	58.77
	CC+ (full, Core)	55.02	56.31	55.83	56.33	56.31
	CC+ (partial, VCG)	49.19	52.77	52.64	52.78	52.76
	CC+ (full, VCG)	46.32	47.29	46.87	47.41	47.40
	iBundle	59.58	56.01	53.84	54.18	54.14

Table 4 Transportation with 25 items and 15 bidders (VCG bidder gain 37.25%).

Result 1 *(Mean efficiency across auction formats and bidder types).* The mean efficiency for the CC and the CC+ auction is higher than 96.9% for all restricted bidder types (Heuristic 5of20, Powerset6, and Powerset10) and all tested value models, except the Real Estate 5x3 value model, where the bidders were interested in a very large number of packages. In the Real Estate 5x3 model, the CC and the CC+ auction yielded an average efficiency of 91.9-94% for restricted bidder types, which is due to the fact that a smaller proportion of the valuations are elicited in larger value models if bidders are restricted to < 10 bids per round. If all bidders follow a powerset strategy, the CC+ auction is almost fully efficient. Small inefficiencies of $< 0.3\%$ in some cases are due to the minimum bid increment. With an ϵ bid increment and m items, the outcome of a CC+ auction without last-and-final bids can be $(m-1)\epsilon$ away from full efficiency. An unrestricted

[3] We applied the nonparametric Wilcoxon rank sum test for testing the difference between the treatments: $\sim$ is used to indicate an insignificant order, $\succ^{*}$ indicates significance at the 10% level, $\succ^{**}$ indicates significance at the 5% level, and $\succ^{***}$ indicates significance at the 1% level.

Measure \ Bidder Type		Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	96.52	98.00	96.99	97.97	99.03
	CC+ (partial)	96.37	99.04	97.47	98.15	100.00
	CC+ (full)	96.47	99.04	97.47	98.15	100.00
	iBundle	100.00	93.92	98.91	99.30	43.02
Min. Efficiency in %	CC	71.85	85.63	80.64	75.18	90.02
	CC+ (partial)	71.85	93.17	80.64	75.18	99.90
	CC+ (full)	71.85	93.17	75.18	75.18	99.90
	iBundle	100.00	81.05	92.74	96.04	14.40
Mean Rounds	CC	288.44	270.04	269.88	268.44	264.36
	CC+ (partial)	291.44	293.64	295.10	291.70	287.38
	CC+ (full)	329.02	299.14	300.00	295.72	294.82
	iBundle	1537.18	19951.04	16176.36	9756.16	1.00
Mean # of Bids	CC	1914.74	5421.78	6397.74	10061.70	80269.48
	CC+ (partial)	1922.86	5484.46	6467.54	10127.52	80337.72
	CC+ (full)	1995.94	5484.72	6465.74	10126.56	80340.70
	iBundle	484560.06	452946.94	4022475.72	414268.22	645.00
Mean Revenue in %	CC	87.12	96.02	94.38	95.71	97.07
	CC+ (partial, Core)	68.02	84.20	75.40	82.21	86.60
	CC+ (full, Core)	67.80	83.46	74.77	81.56	85.91
	CC+ (partial, VCG)	56.68	82.98	71.87	80.45	85.89
	CC+ (full, VCG)	55.84	81.52	70.55	79.45	84.79
	iBundle	86.07	81.46	83.72	84.14	0.00

Table 5 Real Estate 3x3 with 9 items and 5 bidders (VCG bidder gain 15.31%).

Measure \ Bidder Type		Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	83.01	93.80	90.19	92.09	99.29
	CC+ (partial)	83.17	93.93	90.09	91.93	99.87
	CC+ (full)	83.09	93.90	90.09	91.93	99.86
Min. Efficiency in %	CC	60.78	74.32	74.32	74.32	89.61
	CC+ (partial)	60.78	74.32	74.32	74.32	99.07
	CC+ (full)	60.78	74.32	74.32	74.32	99.07
Mean Rounds	CC	42.34	40.98	40.04	39.58	38.40
	CC+ (partial)	42.62	42.64	43.10	42.78	42.58
	CC+ (full)	44.70	43.36	43.66	43.40	43.06
Mean # of Bids	CC	247.74	919.22	1099.54	1780.52	368823.10
	CC+ (partial)	248.10	929.68	1109.38	1795.28	369040.14
	CC+ (full)	251.16	929.74	1109.30	1795.32	369040.14
Mean Revenue in %	CC	78.79	89.67	85.94	88.29	96.93
	CC+ (partial, Core)	64.93	77.47	74.09	76.86	87.50
	CC+ (full, Core)	64.75	77.36	73.99	76.76	87.35
	CC+ (partial, VCG)	59.49	70.16	67.88	70.41	84.23
	CC+ (full, VCG)	58.25	69.90	67.82	70.23	83.89

Table 6 Real Estate 5x3 with 15 items and 5+1 bidders (VCG bidder gain 15.5%).

powerset strategy in the CC auction leads to 96.8% efficiency on average for all value models that we analyzed, illustrating the robustness of this simple auction format.

iBundle achieves full efficiency with straightforward bidders as predicted by the theory. With heuristic 5of20, powerset6, and powerset10 bidders the average efficiency results are in most instances significantly worse, but in the Real Estate 3x3 value model also better than the CC+ auction.

In the following, we refer to the Real Estate 5x3 value model as a large value model, because bidders are interested in 443 or even 32,767 packages. All other value models are referred to as small. Note that in realistic applications, we do not expect bidders to have several hundred or thousands of positive valuations for packages, and the "small" value models describe realistic problem sizes with up to 129 packages with a positive valuation.

Result 2 *(Efficiency of the CC and the CC+ auction in small and large value models).* In the small value models, the CC+ auction achieves significantly higher efficiency than the CC auction (CC+ $\succ^{***}$ CC or CC+ $\succ^{**}$ CC, depending on the value model and on the type of the powerset bidder). In the large RealEstate 5x3 value model with powerset bidders, the CC+ auction has significantly higher efficiency (CC+ $\succ^{***}$ CC), but there are no significant differences for restricted bidding strategies.

In small value models, powerset6 and powerset10 bidders reveal a larger proportion of their valuations, which has a positive effect on the efficiency. In the larger RealEstate 5x3 value model, a smaller proportion of the valuations are revealed in each round and the advantages of the CC+ auction compared to the CC auction vanish. Note that even for such a large value model, the mean efficiency is around 92% even for powerset bidders, who are restricted to six or ten bids per round, and almost 94% for heuristic 5of20 bidders.

From our theoretical treatment, we know that the efficiency of the CC auction can be almost 0% in the worst case. In the following, we take a look at the lowest efficiency, which has been achieved in experiments with different CATS value models.

Result 3 *(Minimum efficiency for restricted and unrestricted powerset bidders).* In Airport, Transportation and Pairwise Synergy value models, the CC auction and iBundle have significantly lower minimum efficiency than the CC+ auction for restricted and unrestricted powerset bidders (see also Appendix D). In the Real Estate 3x3 and 5x3 value models, the minimum efficiency goes down to 74% for restricted bidders. With powerset bidders, the minimum efficiency in the CC auction, was always significantly lower than in the CC+ auction which was almost fully efficient also in the worst case (CC+ $\succ^{***}$ CC).

Result 4 *(Number of rounds and bids).* The difference in the number of rounds and bids between CC and CC+ is always significant (CC+ $\succ^{***}$ CC), but rather small. Note that the number of rounds of a CC+ auction with full revelation price update rule is not necessarily higher than in the CC+ auction with partial revelation price update rule, since prices on more items are increased by the CC+ auction with full revelation price update rule. The number of rounds in iBundle is orders of magnitude higher than in the CC or the CC+ auction. With powerset bidders iBundle terminates prematurely, which can result in low revenue and number of rounds, as for example in the Real Estate 3x3 value model. In this value model, there were several thousand auction rounds in iBundle.

We always used a minimum bid increment of 1. Clearly, the number of auction rounds can be decreased by increasing the bid increment, but at the expense of efficiency. Note that the valuations in the Real Estate 3x3 value model were determined in a very different way to the Real Estate 5x3 model, as was explained in Section 5.1. The valuations for items and packages were on different levels, leading to a different number of auction rounds and a different number of bids submitted.

Result 5 *(Average revenue).* The average auctioneer revenue is the highest in the CC auction and decreases significantly with the introduction of bidder-Pareto-optimal core prices and even more so with VCG prices (CC $\succ^{***}$ CC+ (Core) $\succ^{***}$ CC+ (VCG)). The revenue of iBundle with straightforward bidders is significantly higher than that of the CC+ auction with powerset bidders and a VCG rule.

Note that iBundle is always in the core with straightforward bidders, while the VCG mechanism is not, which can lead to lower revenue if valuations are not buyer submodular.

5.3. Summary

As theory predicts, iBundle is fully efficient in the computational experiments with straightforward bidders, and so is the CC+ auction with powerset bidders. This full efficiency comes at a cost in both auction formats. The number of auction rounds in the CC+ auction is only slightly increased compared to the CC auction, but the number of bids submitted by powerset bidders was much higher, in particular with the large Real Estate 5x3 value model. Note that the number of bids revealed in iBundle with straightforward bidders was an order of magnitude higher than the number of bids submitted by a fully efficient CC+ auction with powerset bidders in all value models. For example, in the Transportation value model, the fully efficient CC+ auction led to 825.44 bids on average, whereas iBundle led to 7785,48 bids. This was even worse in the case of the Real Estate 3x3 value model (80,340.70 in CC+ vs. 484,560.00 bids on average in iBundle), and the Real Estate 5x3 value model, where more than 4.8 million bids were submitted in the experiments that we run.

If bidders are not able to follow such equilibrium strategies, either for the number of rounds or the number of bids that need to be submitted, and are restricted in the number of bids submitted in each round, full efficiency can no longer be reached. To gain an understanding of how such restrictions impact the efficiency, we have run simulations with the heuristic 5of20, powerset6, and powerset10 bidders. Interestingly, the auctions still yield fairly high levels of efficiency on average, mostly higher than 90%. Note, however, that the number of rounds and the number of bids submitted in iBundle is much higher than in the CC or the CC+ auctions. In most applications with human bidders, more than fifty auction rounds would not be acceptable, and the auctioneer would have to increase the minimum bid increment significantly in iBundle, which can lead to additional inefficiencies.

6. Conclusions

Combinatorial auctions have led to a substantial amount of research and found a number of applications in high-stakes auctions for industrial procurement, logistics, energy trading, and the sale of spectrum licenses. Anonymous linear ask prices are very desirable and sometimes even essential for many of these applications (Meeus et al. 2009). Unfortunately, Walrasian equilibria with linear prices are only possible for restricted valuations. Already Kelso and Crawford (1982) showed that the "goods are substitutes" property (aka gross substitutes) is a sufficient and an almost necessary condition for the existence of linear competitive equilibrium prices. Later, Gul and Stacchetti (2000) found that even if bidders' valuation functions satisfy the restrictive "goods are substitutes" condition, no ascending VCG auction exists that uses anonymous linear prices. Bikhchandani and Ostroy (2002) show that personalized non-linear competitive equilibrium prices always exist. Several auction designs are based on these fundamental theoretical results and use non-linear personalized prices. While these NLPPAs achieve efficiency, they only satisfy an ex-post equilibrium if the valuations meet buyer submodularity conditions, and they lead to a very large number of auction rounds requiring bidders to follow the straightforward strategy throughout.

These theoretical results assume ask prices throughout the auction to be equivalent to the final competitive equilibrium prices and the payments of bidders. The CC auction differentiates, which is also a way around the negative theoretical results. Still, the CC auction (Porter et al. 2003) cannot be fully efficient. We provide worst-case bounds on the efficiency of the CC auction with straightforward bidders, and propose an extension of the CC auction, the CC+ auction design, which achieves full efficiency with bidders following a powerset strategy. This design modifies the price update rule of the CC auction and adds a VCG payment rule. We show that with such a VCG payment rule, a powerset strategy leads even to an ex-post equilibrium. Note that there are no restrictions on the type of valuations of bidders, which is important for any application. The discussion also shows that the number of ask prices that need to be communicated by the auctioneer, as well as the number of bids required by bidders, is significantly lower than in NLPPAs.

Clearly, a powerset strategy is prohibitive for any but small combinatorial auctions and some other auction rules of the CC+ auction are impractical for real world applications. Actually, the

CC+ auction is almost equivalent to a VCG auction, except that bidders learn the highest valuations of items throughout the auction, which they do not in a sealed-bid auction. Since the CC+ auction is iterative, however, we step back from dominant strategies and limit ourselves to an ex-post equilibrium. This is in line with previous results on the VCG auction (Green and Laffont 1979, Holmstrom 1979), which show that any efficient mechanism with the dominant strategy property are equivalent to the VCG mechanism, always leading to identical equilibrium outcomes. Later, Williams (1999) found that all Bayesian mechanisms that yield efficient equilibrium outcomes and in which losers have zero payoffs lead to the same expected equilibrium payments as the VCG mechanism. So it is not surprising that the CC+ auction also uses a VCG payment rule to satisfy an ex-post equilibrium.

The CC+ auction is of theoretical and practical relevance, as it shows under which conditions full efficiency with a strong solution concept for general valuations is possible with a clock auction. We can run sensitivity analyses to investigate how robust the CC+ auction is against deviations from the equilibrium strategies. Interestingly, even if the number of bids submitted in each round is severely restricted or bidders heuristically select some of their "best" bids in each round, both the CC and the CC+ auction achieve very high efficiency levels. The results also explain some of the high efficiency and robustness results of the CC auctions in the lab. However, we also show that the efficiency decreases with an increase in the number of packages of interest to bidders, which can be explained by communication complexity being a fundamental problem in all combinatorial auctions (Nisan and Segal 2001).

References

Adomavicius, G., A. Gupta. 2005. Towards comprehensive real-time bidder support in iterative combinatorial auctions. *Information Systems Research (ISR)* **16** 169–185.

An, N., W. Elmaghraby, P. Keskinocak. 2005. Bidding strategies and their impact on revenues in combinatorial auctions. *Journal of Revenue and Pricing Management* .

Arrow, K. J., G. Debreu. 1954. Existence of an equilibrium for competitive economy. *Econometrica* **22** 265–290.

Ausubel, L., P. Cramton, P. Milgrom. 2006. The clock-proxy auction: A practical combinatorial auction design. P. Cramton, Y. Shoham, R. Steinberg, eds., *Combinatorial Auctions*. MIT Press, Cambridge, MA.

Ausubel, L., P. Milgrom. 2002. Ascending auctions with package bidding. *Frontiers of Theoretical Economics* **1** 1–42.

Ausubel, L., P. Milgrom. 2006a. Ascending proxy auctions. P. Cramton, Y. Shoham, R. Steinberg, eds., *Combinatorial Auctions*. MIT Press, Cambridge, MA.

Ausubel, L., P. Milgrom. 2006b. The lovely but lonely vickrey auction. P. Cramton, Y. Shoham, R. Steinberg, eds., *Combinatorial Auctions*. MIT Press, Cambridge, MA.

Banks, J., M. Olson, D. Porter, S. Rassenti, V. Smith. 2003. Theory, experiment and the fcc spectrum auctions. *Journal of Economic Behavior and Organization* **51** 303–350.

Bapna, R., S. Das, R. Garfinkel, J. Stallaert. 2007. A market design for grid computing. *INFORMS Journal on Computing* **20** 100–111.

Bapna, R., C. Dellarocas, S. Rice. 2010. Vertically differentiated simultaneous vickrey auctions: Theory and experimental evidence. *Management Science* **56** 1074–1092.

Bichler, M., A. Davenport, G. Hohner, J. Kalagnanam. 2006. Industrial procurement auctions. P. Cramton, Y. Shoham, R. Steinberg, eds., *Combinatorial Auctions*. MIT Press.

Bichler, M., A. Gupta, W. Ketter. 2010. Designing smart markets. *Information Systems Research* **to appear**(1).

Bichler, M., P. Shabalin, A. Pikovsky. 2009. A computational analysis of linear-price iterative combinatorial auctions. *Information Systems Research* **20**(1) 33–59.

Bikhchandani, S., J. W. Mamer. 1997. Competitive equilibrium in an exchange economy with indivisibilities. *Journal of Economic Theory* **74** 385–413.

Bikhchandani, S., J. M. Ostroy. 2002. The package assignment model. *Journal of Economic Theory* **107**(2) 377–406.

Blumrosen, L., N. Nisan. 2007. *Combinatorial Auctions*. Cambridge University Press.

Brunner, Ch., J. K. Goeree, Ch. Hold, J. Ledyard. 2009. An experimental test of flexible combinatorial spectrum auction formats. *American Economic Journal: Micro-Economics* **forthcoming**.

Caplice, Y., C.and Sheffi. 2006. Combinatorial auctions for truckload transportation. P. Cramton, Y. Shoham, R. Steinberg, eds., *Combinatorial Auctions*. MIT Press, Cambridge, MA.

Conen, W., T. Sandholm. 2001. Preference elicitation in combinatorial auctions. *ACM Conference on Electronic Commerce*. Tampa, FL, 256–259.

Conitzer, V., T. Sandholm. 2002. Vote elicitation: Complexity and strategy-proofness. *AAAI*. 392–397.

Cramton, P. 2009. *Auctioning the Digital Dividend*. Karlsruhe Institute of Technology.

Cramton, P., Y. Shoham, R. Steinberg, eds. 2006a. *Combinatorial Auctions*. MIT Press, Cambridge, MA.

Cramton, P., Y. Shoham, R. Steinberg. 2006b. Introduction to combinatorial auctions. P. Cramton, Y. Shoham, R. Steinberg, eds., *Combinatorial Auctions*. MIT Press, Cambridge, MA.

Day, R., P. Milgrom. 2007. Core-selecting package auctions. *International Journal of Game Theory* .

Day, R., S. Raghavan. 2007. Fair payments for efficient allocations in public sector combinatorial auctions. *Management Science* **53** 1389–1406.

de Vries, S., J. Schummer, R. Vohra. 2007. On ascending vickrey auctions for heterogeneous objects. *Journal of Economic Theory* **132** 95–118.

Demange, G., D. Gale, M. Sotomayor. 1986. Multi-item auctions. *Journal of Political Economy* **94** 863–872.

Gallien, J., L. Wein. 2005. A smart market for industrial procurement with capacity constraints. *Management Science* **51** 76–91.

Georgescu-Roegen, N. 1979. Methods in economic science. *Journal of Economic Issues* **13** 317–328.

Goeree, Jacob K., Charles A. Holt. 2008. Hierarchical package bidding: A paper & pencil combinatorial auction. *Games and Economic Behavior* **to appear** –. doi:10.1016/j.geb.2008.02.013. URL `http://www.sciencedirect.com/science/article/B6WFW-4S6G912-1/2/7d2d8a7a340d0bfeca617e24796c49de`.

Green, J., J. Laffont. 1979. *Incentives in public decision making*. North Holland, Amsterdaml.

Green, J., J.-J. Laffont. 1977. Characterization of satisfactory mechanisms for the revelation of preferences for public goods. *Econometrica* **45** 427–438.

Greenwald, A., K. Kannan, R. Krishnan. 2010. On evaluating information revelation policies in procurement auctions. *Information Systems Research* **21** 15–37.

Gul, F., E. Stacchetti. 1999. Walrasian equilibrium with gross substitutes. *Journal of Economic Theory* **87** 95–124.

Gul, F., E. Stacchetti. 2000. The english auction with differentiated commodities. *Journal of Economic Theory* **92** 66–95.

Guo, Z., G.J. Koehler, A. B. Whinston. 2007. A market-based optimization algorithm for distributed systems. *Management Science* **53** 1345–1358.

Holmstrom, B. 1979. Groves' scheme on restricted domains. *Econometrica* **47** 1137–1144.

Hu, X., Z. Lin, A. Whinston, H. Zhang. 2004. Hope or hype: On the viability of escrow services as trusted third parties in online auction environments. *Information Systems Research* **15** 236–249.

Hurwicz, L. 1977. On the dimensional requirements of informationally decentralized pareto-satisfactory processes. K.J. Arrow, L. Hurwicz, eds., *Studies in Resource Allocation Processes*. Cambridge University Press, New York, USA.

Isaac, M., T. Salmon, A. Zillante. 2007. A theory of jump bidding in ascending auctions. *Journal of Economic Behaviour and Organization* **62** 144–164.

Jordan, J. S. 1982. The competitive allocation process is informationally efficient uniquely. *Journal of Economic Theory* **28**(1) 1–18.

Kagel, J. H., Y. Lien, P. Milgrom. 2009. Ascending prices and package bids: An experimental analysis. *AEA Conference*.

Kelso, A. S., V. P. Crawford. 1982. Job matching, coalition formation , and gross substitute. *Econometrica* **50** 1483–1504.

Kwasnica, T., J. O. Ledyard, D. Porter, C. DeMartini. 2005. A new and improved design for multi-objective iterative auctions. *Management Science* **51**(3) 419–434.

Leyton-Brown, K., M. Pearson, Y. Shoham. 2000. Towards a universal test suite for combinatorial auction algorithms. *ACM Conference on Electronic Commerce*. 66–76.

Liu, D., J. Chen, A. Whinston. 2010. Ex ante information and the design of keyword auctions. *Information Systems Research* **21** 133–154.

McCabe, K., S. Rassenti, V. Smith. 1991. Smart computer-assisted markets. *Science* **254** 534–538.

Meeus, L., K. Verhaegen, R. Belmans. 2009. Block order restrictions in combinatorial electric energy auctions. *European Journal of Operational Research* **196** 1202–1206.

Milgrom, P. R., R. J. Weber. 1982. A theory of auctions and competitive bidding. *Econometrica* **50**(5) 1089–1122.

Mishra, D., D. Parkes. 2007. Ascending price vickrey auctions for general valuations. *Journal of Economic Theory* **132** 335–366.

Mount, K., S. Reiter. 1974. The information size of message spaces. *Journal of Economic Theory* **28** 1–18.

Nisan, N., I. Segal. 2001. The communication complexity of efficient allocation problems. *DIMACS workshop on Computational Issues in Game Theory and Mechanism Design*. Minneapolis, MI.

Nisan, N., I. Segal. 2006. The communcation requirements of efficient allocations and supporting prices. *Journal of Economic Theory* **129** 192–224.

Parkes, D. 2006. Iterative combinatorial auctions. P. Cramton, Y. Shoham, R. Steinberg, eds., *Combinatorial Auctions*. MIT Press, Cambridge, MA.

Parkes, D., L. H. Ungar. 2000. Iterative combinatorial auctions: Theory and practice. *17th National Conference on Artificial Intelligence (AAAI-00)*.

Porter, D., S. Rassenti, A. Roopnarine, V. Smith. 2003. Combinatorial auction design. *Proceedings of the National Academy of Sciences of the United States of America (PNAS)* **100** 11153–11157.

Rassenti, S., V. L. Smith, R. L. Bulfin. 1982. A combinatorial auction mechanism for airport time slot allocations. *Bell Journal of Economics* **13** 402–417.

Rothkopf, M. H. 2007. Thirteen reasons why the vickrey-clarke-groves process is not practical. *Operations Research* **55** 191–197.

Rothkopf, M. H., R. M. Harstad. 1994. Modeling competitive bidding: a critical essay. *Management Science* **40** 364–84.

Sandholm, Levine D. Concordia M. Martyn P. Hughes R. Jacobs J., T., D. Begg. 2006. Changing the game in strategic sourcing at procter & gamble: Expressive competition enabled by optimization. *Interfaces* **36**(1) 55–68.

Scheffel, T., A. Pikovsky, M. Bichler, K. Guler. 2010. An experimental comparison of linear and non-linear price combinatorial auctions. *Information Systems Research* **to appear**.

Schneider, S., P. Shabalin, M. Bichler. 2010. On the robustness of non-linear personalized price combinatorial auctions. *European Journal on Operational Research* **to appear**.

Williams, S. R. 1999. A characterization of efficient, bayesian incentive compatible mechanisms. *Journal of Economic Theory* **14** 155–180.

Xia, Mu, Koehler G. J., Whinston A. B. 2004. Pricing combinatorial auctions. *European Journal of Operational Research* **154**(1) 251–270.

Appendix A: Proofs

Theorem 2:If bidder valuations are demand masking and all bidders follow the straightforward strategy in the CC auction, then the efficiency converges to $\frac{2}{m+1}$ in the worst case.

Proof: The following proof is provided for two or more items for sale and $2m-1$ bidders. With less than $2m-1$ bidders and XOR bidding, efficiency can only increase. Without loss of generality, we assume item-level bid increments of $\epsilon = 1$ in each round $t \in \mathcal{T} \subset \mathbb{N}$. We consider the value μ as given and determine ξ and ν_h such that efficiency decreases to the worst case of 0%.

Case a) $\mu \geq \xi + \sum_h \nu_h$:

The efficient solution is to sell $R \cup S_h$ to one of the bidders h_a or h_b. The CC auction terminates with the efficient outcome in this case.

Case b) $\mu < \xi + \sum_h \nu_h \wedge \xi = \mu$:

The proof is by showing that a straightforward bidder h_a cannot bid on S_h throughout the auction in a demand masking set of valuations. For this, the payoff $\pi_{h_a}(R \dot\cup S_k)$ must be higher than $\pi_{h_a}(S_h)$ for each bidder h_a in each round of the auction $t \in \mathcal{T}$:

$$v_{h_a}(R \dot\cup S_h) - \beta_{h_a,t}(R \dot\cup S_h) > v_{h_a}(S_h) - \beta_{h_a,t}(S_h) \quad \forall h \in \{2, ..., g\}, \forall t \in \mathcal{T} \tag{1}$$

Since we know that $v_{h_a}(R \dot\cup S_h) = v_{h_b}(R \dot\cup S_h) = \mu$, and all bidders bid straightforward, we know that the price for all the items in $\mathcal{K}$ rises in each round by ϵ. Therefore, inequality (1) can be rewritten as

$$\mu - |R \dot\cup S_h| t\epsilon > \nu_h - |S_h| t\epsilon \overset{\epsilon=1}{\Longrightarrow} t < \frac{\mu - \nu_h}{|R|} \quad \forall h \in \{2, ..., g\}, \forall t \in \mathcal{T} \tag{2}$$

Inequality (2) shows that as long as t is smaller than the right-hand side, a straightforward bidder always bids on the package $R \dot\cup S_h$. We can now determine a round $t_{min} = \min\{t | t \geq \frac{\mu - \nu_h}{|R|}, \forall h_a\}$, in which the payoff $\pi_{h_a}(R \dot\cup S_h)$ is for the first time smaller or equal to the payoff $\pi_{h_a}(S_h)$. We call t_{min} the *decisive round.* If either the right side or both sides of inequality (1) become negative in round t_{min}, bidder h_a cannot bid on S_h or the auction ends for bidder h_a as also the ask price for $R \dot\cup S_h$ is higher than $v_{h_a}(R \dot\cup S_h)$. If straightforward bidder h_a does not reveal his preferences for S_h throughout the auction, then the auctioneer in a class A auction selects any of the other bids with a revenue of μ, resulting in an efficiency of $\mu/(\xi + \sum_h \nu_h)$.

We determine maximal ν_h such that in round t_{min} the payoff of bidder h_a on package S_h is negative, which minimizes efficiency. We know that as long as bidder h_a's payoff is negative in the decisive round t_{min}, i.e., $\nu_h - |S_h| t_{min} < 0$, then bidder h_a does not bid on S_h. We also know that $t_{min} = \lceil (\mu - \nu_h)/|R| \rceil$ is the decisive round. We can now maximize ν_h such that $\nu_h - |S_h| \lceil (\mu - \nu_h)/|R| \rceil < 0$, resulting in $\nu_{h_{max}} = \max\{\nu_h | \nu_h < |S_h| \mu / (|R| + |S_h|)\}$. In order to maximize $\sum_h \nu_h$ and so minimize the efficiency $\mu/(\xi + \sum_h \nu_h)$, we set $|R| = 1$ and $|S_h| = 1$ for all $h \in \{2, ..., g\}$. This results in an efficiency of $E(X) = \mu/(\xi + \sum_h (\frac{\mu}{2} - \rho))$ with $\rho > 0$. With $\rho \to 0$ and $\xi = \mu$ efficiency decreases to $2/(g+1)$, which is $2/(m+1)$ in the worst case. Note that it does not matter if ξ is smaller or larger than $\sum_h \nu_h$.

Case c) $\mu < \xi + \sum_h \nu_h \wedge \mu \neq \xi$:

Efficiency can only increase compared to case b) considering the worst case. Either the enumerator of $E(X) = \frac{\max\{\xi, \mu\}}{\max\{\xi + \sum_h \nu_h, \mu + \sum_h \nu_h\}}$ increases or the denominator decreases.

- $\xi > \mu$: $\Rightarrow E(X) = \frac{\xi}{\xi + \sum_h \nu_h} = \frac{\mu + \delta}{\mu + \delta + \sum_h \nu_h}$ with $\delta > 0$ is always greater than the efficiency $E(X)$ in case b).
- $\xi < \mu$: $\Rightarrow$
  - —either $E(X) = \frac{\mu}{\xi + \sum_h \nu_h}$ which is greater than $E(X) = \frac{\mu}{\mu + \sum_h \nu_h}$ the efficiency of case b).
  - —or $E(X) = \frac{\mu}{\mu + \sum_{h=2}^{g-1} \nu_h}$ which is also greater than $E(X) = \frac{\mu}{\mu + \sum_{h=2}^{g} \nu_h}$ the efficiency of case b).

□

Proposition 1:If all bidders follow the powerset strategy, the efficiency of the CC auction converges to 0% in the worst case.

Proof: Since efficiency cannot be negative it is sufficient to present an example in which the efficiency is almost 0%. Assume two bidders and three items for sale. The two bidders have valuations for packages as shown in Table 7. They value all other packages with zero. The final ask prices are $\beta_{(1)} = 2$, $\beta_{(2)} = 2$ and $\beta_{(3)} = 1$, and the final allocation assigns package $(1, 2)$ to bidder 1, which is inefficient if $\mu > 4$. Efficiency decreases to 0% if $\mu \to \infty$.

□

	$(1,2)$	$(2,3)$
v_1	4	μ
v_2	2	0

Table 7 Valuations that lead to inefficiencies in the CC auction with assuming powerset bidders.

We assume no free disposal concerning the valuations in Table 7. Otherwise, bidder 1 has a valuation of μ also for package $(1,2,3)$, and this would be sold to bidder 1 for a price of 5. The payoff for bidder 1 in this allocation would be $\mu - 5$, which would be efficient, as the sum of the bidders' payoffs and the auctioneer revenue is maximized. Free disposal can lead to situations in which powerset bidding drives up prices to very high levels and reduces bidders' utility. It can also lead to high inefficiency (see Appendix C). Consequently, powerset bidding is even more unlikely in a CC auction with free disposal.

Corollary 1:If all bidders follow the powerset strategy, the CC auction with the partial revelation price update rule terminates with an efficient outcome.

Proof: Based on the statement of Theorem 1, we only need to show that the valuations of relevant packages are revealed with powerset bidders in the modified CC auction. Through the construction of the partial revelation price update rule, powerset bidders who are not part of the efficient allocation reveal all their valuations. But the rule also ensures that all the bidders in the efficient allocation reveal their valuations on all packages except the ones that are in the winning allocation. As long as a bidder bids on more than one package the auction continues as each bidder can only win one package. As long as a bidder bids on a package that is not winning, prices increase and he can keep bidding. Thus the CC auction with the partial price update rule elicits all valuations except the ones of winning packages and terminates with an efficient allocation. □

Corollary 2:A powerset strategy is an ex-post equilibrium in the CC+ auction.

Proof: The proof for the ex-post equilibrium strategy is from the VCG mechanism. Let t_j denote the type of bidder j. We look at the bidder j and assume all other bidders follow the truth revealing powerset strategy. Bidder j receives a payment of $\sum_{i\neq j} u_i(t'_i, X) - \sum_{i\neq j} u_i(t'_i, X_{-j})$ from the center. The final payoff to bidder j reporting type t' and an allocation X and a VCG payment rule is $u_j(t_j, X) + \sum_{i\neq j} u_i(t'_i, X) - \sum_{i\neq j} u_i(t'_i, X_{-j})$. A bidder in this payment rule cannot affect the choice of X_{-j}. Hence, j can focus on maximizing $u_j(t_j, X) + \sum_{i\neq j} u_i(t'_i, X)$, i.e., his utility and the sum of the other's utilities. As the auction will maximize $\sum_i u_i(t'_i, X)$, j's utility will be maximized, if $t'_j = t_j$. □

Proposition 2:Powerset bidding does not satisfy an ex-post equilibrium in the CC+ auction with only a partial revelation price update rule.

Proof: In the example in Table 8, the CC+ auction with a partial revelation price update rule ends up with final ask prices of $\beta_{(1)} = 3$ and $\beta_{(2)} = 4$, before the VCG prices are calculated. If the auctioneer calculates VCG prices based on the submitted bids, then bidder 2 pays $3 - (7 - 5) = 1$ for the item (1). If bidder 2 knew $v_3(2)$, he could have bid up to 6 on item (2). This would increase the final ask price for (2) to 7, and lead to a new VCG price of $3 - (10 - 7) = 0$ for (1) for bidder 2. In a VCG mechanism, bidder 2 could not influence the bid submission of bidder 3 in a similar way, which is why the VCG mechanism has a dominant strategy. Therefore, in the CC+ auction with a partial revelation price update rule, the strategy of bidder 2 is not independent of other bidders' types. Even if the other bidders bid truthfully, a bidder could improve his payoff by deviating from a truth revealing powerset strategy, if he knew the other bidders' types and the other bidders truthfully follow the powerset strategy.

□

Corollary 3:The CC+ auction with powerset bidders terminates with a core outcome if it charges bidder-Pareto-optimal prices as payments instead of VCG prices.

Proof: Since the CC+ auction elicit all valuations from all bidders and the algorithm from Day and Raghavan (2007) calculates core prices upon the submitted bids the statement is shown. □

	(1)	(2)
v_1	0	3
v_2	3*	0
v_3	2	7*

Table 8 **Valuations that do not lead to an ex-post equilibrium with powerset bidders when using the partial revelation price update rule in the CC+ auction.**

Appendix B: Ex-Post Equilibrium of the CC+ Auction

Does the CC+ auction satisfy a dominant strategy or an ex-post equilibrium? In the single-unit case, there has been an interesting recent discussion on the types of ascending auctions that actually satisfy a dominant strategy equilibrium. Isaac et al. (2007) have shown that while the clock version of an ascending single-item auction has a dominant strategy, the widespread English auction, which allows jump bids, has not.

The CC+ auction can be seen as a multi-item generalization of the ascending clock auction. Also, the VCG auction can be thought of as a single-round version of the CC+ auction, in which the bidder's dominant strategy is to bid truthfully on all possible packages, similar to a powerset strategy. Both auctions satisfy a dominant strategy equilibrium. Does the CC+ auction also satisfy a dominant strategy, or is it restricted to an ex-post equilibrium? In the following, we provide an example in which signals revealed throughout the CC+ auction can make it beneficial for a bidder to deviate from his truth-telling powerset strategy when also others deviate from this strategy.

	(1)	(2)	(1, 2)
v_1	2*	0	0
v_2	0	3*	0
v_3	0	0	4

Table 9 **Example of the difference between the VCG auction and the CC+ auction.**

The valuations for three bidders and two items are given in Table 9. The VCG price of bidder 1 is $2-(5-4)=1$ for item (1), and his payoff is 1. Now, assume that bidder 1 knows that bidder 2 will increase his bid on (2) to 4, if the ask price for (1) was 3. In round 2, the price clock ticks to 2 for each item and all three bidders signal demand at these prices. In round 3, prices are 3 for both items and again bidders 1 and 2 will signal demand. This will encourage bidder 2 to signal demand even in round 4 for item (2), when bidder 1 drops out. Now, bidder 1 gets a VCG price of $3+(7-4)=0$ and consequently increased his true payoff from 1 to 2. Bidder 2 learns through the course of the CC+ auction that there is a demand for (1) at a price of 3, which would not be possible in a direct revelation VCG auction.

This cannot happen in a clock auction with only a single item, as the bidders can only drop out or continue to signal demand on a single item. This illustrates that the dominant strategy equilibrium does not extend from the single-item clock auction to its multi-item generalization. The powerset strategy in a multi-item CC+ auction is therefore an ex-post equilibrium and not a dominant strategy equilibrium.

Appendix C: Powerset Strategies in a CC Auction with Free Disposal

In the following, we describe an economy with powerset bidders and free disposal. We show that the CC auction leads to very high prices, thus reducing the bidders' utility, even in cases where there is no competition. The example shows that the inefficiency in these situations can be almost as low as 50%.

Given the valuations in Table 10 and an economy without free disposal, the bidders would all bid on a single item only, and the CC auction would stop after the first round at a price of the minimum bid increment ϵ. In an example, assume $m=n=100$, $\epsilon=1$, and $\mu \geq 200$. The allocation assigning bidder i item (i) is efficient and would maximize overall welfare. Bidder 1 would get a payoff of $\pi-1$, while all other bidders achieve a payoff of $(\mu/100)-1$. With an auctioneer revenue of 100, the social welfare is $199\pi/100$. If we assume m is the number of items, then the social welfare would be maximized at $(2m-1)\mu/m$.

Now, with free disposal, bidder 1 would bid on all $2^{(m-1)}$ packages that enfold the item (1) in each round until a price of μ/m is reached and he wins all items. His payoff would be 0 and the auctioneer would make a revenue of μ, which is inefficient. With $m \to \infty$ efficiency converges to 50%.

	(1)	(2)	...	(m)
v_1	μ	0	...	0
v_2	0	$(\mu/m)-\epsilon$	...	0
...	...	...	...	...
v_m	0	0	...	$(\mu/m)-\epsilon$

Table 10 Valuations in an economy with powerset bidders and free disposal.

Appendix D: Further Computational Experiments

D.1. Value Models

We also test a ***Transportation Large*** value model with 50 items and 30 bidders. The other characteristics are as described in Section 5.

The ***Airports*** value model is an implementation of the *matching* scenario from CATS. It models the four largest airports in the USA, each having a predefined number of departure and arrival time slots. For simplicity there is only one slot for each time unit and airport available. Each bidder is interested in obtaining one departure and one arrival slot (i.e., item) in two randomly selected airports. His valuation is proportional to the distance between the airports and reaches maximum when the arrival time matches a certain randomly selected value. The valuation is reduced if the arrival time deviates from this ideal value, or if the time between departure and arrival slots is longer than necessary.

The ***Pairwise Synergy*** value model from An et al. (2005) is defined by a set of valuations of individual items $v(k)$ with $k \in \mathcal{K}$ and a matrix of pairwise item synergies $\{syn_{k,l} : k,l \in \mathcal{K}, syn_{k,l} = syn_{l,k}, syn_{k,k} = 0\}$. The valuation of a package S is then calculated as

$$v(S) = \sum_{k=1}^{|S|} v(k) + \frac{1}{|S|-1} \sum_{k=1}^{|S|} \sum_{l=k+1}^{|S|} syn_{k,l}(v(k)+v(l))$$

A synergy value of 0 corresponds to completely independent items, and the synergy value of 1 means that the package valuation is twice as high as the sum of the individual item valuations. The model is very generic, as it allows different types of synergistic valuations, but it was also used to model valuations in transportation auctions (An et al. 2005). We use the Pairwise Synergy value model with seven items; item valuations are drawn for each auction independently from a uniform distribution between 4 and 12. The synergy values are drawn from a uniform distribution between 1.5 and 2.0. The auctions with the Pairwise Synergy value model have five bidders each. We use a high synergy and low synergy setting.

In the *Real Estate* and *Pairwise Synergy* value models, bidders are interested in a maximum package size of 3, because in these value models large packages are always valued more highly than small ones. This is also motivated by real-world observations (An et al. 2005), in which bidders typically have an upper limit on the number of items they are interested in. Without this limitation, the auction easily degenerates into a scenario with a single winner for the package containing all items.

Measure	Bidder Type	Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	98.48	97.94	98.04	97.86	98.07
	CC+ (partial)	98.19	99.17	99.25	99.33	99.27
	CC+ (full)	98.11	99.14	99.19	99.29	99.26
	iBundle	100.00	86.95	94.22	94.74	95.89
Min. Efficiency in %	CC	90.74	85.00	85.00	85.00	85.00
	CC+ (partial)	90.74	95.29	95.29	95.29	95.29
	CC+ (full)	90.74	96.10	95.35	96.47	96.13
	iBundle	100.00	69.62	76.03	76.40	84.24
Mean Rounds	CC	17.80	17.08	14.00	13.82	13.62
	CC+ (partial)	16.90	14.90	14.86	14.60	14.40
	CC+ (full)	21.48	18.14	18.14	18.00	17.88
	iBundle	57.18	1222.42	860.08	534.06	58.14
Mean # of Bids	CC	234.04	376.20	406.68	496.66	1201.30
	CC+ (partial)	226.76	375.90	407.26	496.76	1201.30
	CC+ (full)	227.40	373.66	404.74	494.50	1199.88
	iBundle	34968.16	35673.06	31686.36	30947.04	25136.04
Mean Revenue in %	CC	83.80	89.11	89.27	89.21	89.32
	CC+ (partial, Day)	67.62	69.97	69.95	70.28	70.39
	CC+ (full, Day)	67.81	69.43	69.34	69.76	69.82
	CC+ (partial, VCG)	56.30	60.70	60.95	61.54	61.68
	CC+ (full, VCG)	54.67	57.78	58.33	58.69	58.72
	iBundle	76.92	64.32	64.91	65.43	66.99

Table 11 Transportation Large with 50 items and 30 bidders (VCG bidder gain 37.25%).

Measure	Bidder Type	Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	98.62	97.18	97.23	97.10	97.17
	CC+ (partial)	98.60	98.74	98.71	98.71	98.71
	CC+ (full)	98.74	98.75	98.62	98.76	98.73
	iBundle	100.00	95.50	98.93	98.13	97.87
Min. Efficiency in %	CC	95.39	92.63	93.42	93.57	93.73
	CC+ (partial)	95.04	96.57	96.57	97.06	96.86
	CC+ (full)	96.10	97.06	95.74	97.06	96.86
	iBundle	100.00	88.50	96.57	93.00	92.55
Mean Rounds	CC	10.94	8.48	8.28	8.20	8.20
	CC+ (partial)	13.58	11.80	11.40	11.28	11.36
	CC+ (full)	17.82	11.86	11.48	11.46	11.42
	iBundle	27.30	48.98	41.58	33.82	33.68
Mean # of Bids	CC	472.72	721.44	786.74	957.68	993.40
	CC+ (partial)	511.22	758.20	819.98	989.44	1025.66
	CC+ (full)	527.80	734.16	798.52	969.44	1004.22
	iBundle	6364.38	6262.72	5522.90	6364.66	7095.52
Mean Revenue in %	CC	82.75	91.30	91.33	91.67	91.75
	CC+ (partial, Core)	41.02	44.45	43.91	44.24	44.09
	CC+ (full, Core)	41.43	43.31	42.90	43.36	43.13
	CC+ (partial, VCG)	35.17	38.61	38.54	38.91	38.64
	CC+ (full, VCG)	36.35	38.04	38.08	38.54	38.38
	iBundle	49.02	48.51	47.88	46.74	47.25

Table 12 Airports with 84 items and 40 bidders (VCG bidder gain 57.76%).

D.2. Experimental Results

Appendix E: List of Symbols

$\mathcal{K}$	set of items
$R, S \subseteq \mathcal{K}$	subset of items
k, l	item index $k, l \in \{1, \dots, m\}$
$\mathcal{I}$	set of bidders
$T \subseteq \mathcal{I}$	subset of bidders
i, j	bidder index $i, j \in \{1, \dots, n\}$
$\mathcal{T}$	set of auction rounds
t	round index $t \in \{1, \dots, r\}$
Γ	set of allocations
X	allocation $X = (X_1, \dots, X_n)$ with package X_i assigned to bidder i
X^*	efficient allocation $X^* = (X_1^*, \dots, X_n^*)$
$\overline{X}$	allocation that maximizes the auctioneers revenue/payoff
$E(X) \in [0, 1]$	efficiency of the allocation X
$v_i(S)$	private valuation of bidder i for package S
$w(T)$	coalitional value of coalition T

Measure \ Bidder Type		Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	99.16	99.06	99.15	99.02	99.29
	CC+ (partial)	99.09	99.83	99.72	99.74	100.00
	CC+ (full)	99.26	99.83	99.72	99.74	100.00
	iBundle	100.00	97.93	99.33	99.14	39.52
Min. Efficiency in %	CC	86.52	94.22	96.32	94.22	94.22
	CC+ (partial)	86.52	97.79	97.20	96.22	99.95
	CC+ (full)	94.53	97.79	97.20	96.22	100.00
	iBundle	100.00	94.19	94.99	93.36	10.90
Mean Rounds	CC	323.66	321.60	325.50	322.94	321.84
	CC+ (partial)	336.72	353.24	356.12	352.24	352.00
	CC+ (full)	367.12	355.56	358.74	355.44	354.52
	iBundle	1596.30	13220.48	11009.38	6821.48	1.00
Mean # of Bids	CC	2068.46	6976.56	8337.60	13368.04	63268.94
	CC+ (partial)	2106.56	7074.98	8440.68	13468.12	63377.04
	CC+ (full)	2165.84	7069.80	8429.24	13463.68	63371.72
	iBundle	332557.40	311450.98	287116.04	303880.26	315.00
Mean Revenue in %	CC	89.76	97.01	96.84	97.20	97.46
	CC+ (partial, Core)	73.42	87.68	83.20	86.81	88.53
	CC+ (full, Core)	72.87	87.06	82.54	86.13	87.95
	CC+ (partial, VCG)	69.93	86.87	81.42	85.71	87.85
	CC+ (full, VCG)	69.33	85.95	80.39	84.71	86.96
	iBundle	88.07	86.20	85.90	87.08	0.00

Table 13 Pairwise Synergy High with 7 items and 5 bidders (VCG bidder gain 12.97%).

Measure \ Bidder Type		Straightforward	5of20	Powerset6	Powerset10	Powerset
Mean Efficiency in %	CC	98.22	98.80	98.44	98.98	99.16
	CC+ (partial)	97.80	99.76	98.96	99.56	100.00
	CC+ (full)	97.76	99.77	98.96	99.56	100.00
	iBundle	100.00	97.66	98.78	99.28	52.88
Min. Efficiency in %	CC	88.28	93.18	91.81	93.18	93.18
	CC+ (partial)	88.28	96.61	91.81	96.26	99.95
	CC+ (full)	88.28	96.61	91.81	96.26	99.95
	iBundle	100.00	93.36	92.76	95.63	22.32
Mean Rounds	CC	368.54	349.38	351.72	350.40	348.78
	CC+ (partial)	395.12	388.88	386.72	386.70	384.04
	CC+ (full)	418.10	390.20	390.98	389.58	388.76
	iBundle	1694.72	13613.30	11696.20	6971.54	1.00
Mean # of Bids	CC	2295.56	7401.86	8800.06	14070.80	68422.96
	CC+ (partial)	2381.44	7519.58	8919.26	14186.08	68541.52
	CC+ (full)	2392.08	7504.12	8903.12	14176.12	68533.64
	iBundle	358160.68	325568.06	304894.88	312606.56	315.00
Mean Revenue in %	CC	88.38	96.85	95.84	96.89	97.23
	CC+ (partial, Core)	73.14	87.11	83.27	86.70	88.26
	CC+ (full, Core)	73.29	86.42	82.34	85.81	87.44
	CC+ (partial, VCG)	69.57	86.29	81.85	85.65	87.69
	CC+ (full, VCG)	68.53	85.16	80.20	84.30	86.55
	iBundle	87.54	83.86	85.09	85.04	0.00

Table 14 Pairwise Synergy Low with 7 items and 5 bidders (VCG bidder gain 13.43%).

Appendix F: List of Abbreviations

(I)CA	(Iterative) Combinatorial Auction
CC	Combinatorial Clock
VCG	Vickrey-Clarke-Groves
NLPPA	Non-Linear-Personalized-Prize Auctions
CE	Competitive Equilibrium