

# An Analysis of Design Problems in Combinatorial Procurement Auctions

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**Management summary:** Traditional auction mechanisms support price negotiations on a single item. The Internet allows for the exchange of much more complex offers in real-time. This is one of the reasons for much research on *multi-dimensional auction mechanisms* allowing negotiations on multiple items, multiple units, or multiple attributes of an item, as they can be regularly found in procurement. Combinatorial auctions, for example, enable suppliers to submit bids on bundles of items. A number of laboratory experiments has shown high allocative efficiency in markets with economies of scope. For suppliers it is easier to express cost savings due to bundling (e.g., decreased transportation or production costs). This can lead to significant savings in total cost of the procurement manager. Procurement negotiations exhibit a number of particularities:

- It is often necessary to consider qualitative attributes or volume discounts in bundle bids. These complex bid types have not been sufficiently analyzed.
- The winner determination problem requires the consideration of a number of additional business constraints, such as limits on the spend on a particular supplier or the number of suppliers.
- Iterative combinatorial auctions have a number of advantages in practical applications, but they also lead to new problems in the determination of ask prices.

In this paper, we will discuss fundamental problems in the design of combinatorial auctions and the particularities of procurement applications.

**Key words:** combinatorial auction, multidimensional auction, industrial procurement, combinatorial optimization.

**Zusammenfassung:** Aus betriebswirtschaftlicher Sicht ist die Anwendung Kombinatorischer Auktionen in der Beschaffung besonders viel versprechend. Die Vorteile umfassen Kosteneinsparungen, die effektive Durchführung komplexer Verhandlungen über mehrere Güter, die Transparenz der Verhandlungen für die

Teilnehmer, Fairness, sowie hohe allokativer Effizienz. Beim Einsatz Kombinatorischer Auktionen kommt eine Reihe grundlegender Entwurfsprobleme zum Tragen. Daneben birgt diese Domäne aber auch eine Reihe spezieller Anforderungen, wie zum Beispiel der Einsatz einer Vielzahl betriebswirtschaftlich motivierter Nebenbedingungen für das Allokationsproblem, sowie die Verwendung alternativer Mehrdimensionaler Gebotstypen.

**Abstract:** Combinatorial auctions are promising auction formats for industrial and public procurement. Potential advantages of using combinatorial auctions include lower overall spend, low transaction costs for multi-item negotiations, fairness and market transparency for suppliers, as well as high allocative efficiency. A number of fundamental design considerations are relevant to the application of combinatorial auctions in procurement. In addition, procurement specialists need to consider several domain-specific requirements, such as additional side constraints as well as alternative multidimensional bid types.

**Keywords:** Combinatorial auction, multidimensional auction, procurement, combinatorial optimization

## 1 Introduction

Procurement negotiations on multiple items or services have typically been conducted as request for quotes (RFQ) or on the phone. Electronic auctions have found increasing adoption in procurement in the past couple of years. They allow for effective price negotiations on single items. Companies such as GlaxoSmithKline use electronic auctions for more than a third of their overall spend [Hann04]. According to a study by the Center for Advanced Purchasing Studies companies use electronic auctions for 5% of their total spend. This proportion is expected to grow in the next few years [BeCC03].

An auction is defined as "a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from the market participants" [McMc87]. The competitive process serves to aggregate the scattered information about bidders' valuations and to dynamically set a price. The auction format determines the rules governing when and how a deal is closed. Klemperer [Klem99] provides a comprehensive introduction to classic auction theory.

Auctions exhibit high allocative efficiency compared to alternative types of negotiations in game-theoretical and experimental analyses [Kage95]. However, auctions have also been criticized in the context of procurement negotiations. Single-item reverse auctions are often considered insufficient for complex procurement negotiations where qualitative attributes of an item, multiple items, or also large quantities of an item are negotiated. Items can be goods or services in this case.

In the past couple of years, several new, "multidimensional" auction mechanisms have been suggested in order to support also complex negotiations. Multidimensional auctions allow for complex bid types, a possibility for economic mechanism design that has become possible on nowadays computer networks. The term is based on dimensions such as quantity and quality of an item that are typically negotiated in procurement [BiKK02]. These types of auctions promise high allocative efficiency even in the presence of complex bidder preferences.

The best known multidimensional auction format is the combinatorial auction, which allows bids on bundles or packages of items [CrSS05]. Game theoretical and empirical analyses of combinatorial auctions are still in their infancy [BaLP89, KrRo96, IsJa00, ?, EwMo03]. Nevertheless, these auctions have been successfully applied in a number of cases. In June 2002 Nigeria has conducted a combinatorial auction for spectrum licenses [KoMM03] and the

US Federal Communications Commission plans their usage in the near future. There are also published cases of applications in industrial procurement and transportation. Examples are the procurement of transportation services at Sears Logistics [LeOP01], the procurement of goods and services at Mars, Incorporated [HoRR03], applications at The Home Depot [ElKe02] and the procurement of school meals in Chile [EpHC02].

Combinatorial auctions address fundamental questions regarding efficiency and prices in complex markets and is based on research results in Economics, Artificial Intelligence, Information Systems, and Operations Research. Information systems play a crucial role as a means to facilitate these types of auctions effectively [WeHN03]. In this article we will discuss essential topics in the design of combinatorial auctions. We will address fundamental design problems and applications in the context of industrial procurement negotiations, which have a number of particularities.

In the next Section, we will provide an overview of various multidimensional auctions and important design goals. Section 3 discusses open and closed combinatorial auction formats. Section 4 gives an overview of applications in industrial procurement. Finally, Section 5 provides a summary of the main findings.

## 2 Auction design with complex bid types

In the simplest case, a multi-item auction is designed to sell multiple identical units of an item. Bids specify price and quantity, as it is common on financial markets with standardized assets. Combinatorial auctions have been discussed in the literature, as they allow negotiations on a set of heterogeneous items. Bidders can specify bundle bids, i.e., a price is defined for a specific subset of the items for auction [CrSS05]. The price is only valid for the entire set and the set is indivisible. For example, a bidder might want to sell 10 units of item  $x$  and 20 units of item  $y$  for a bundle price of €100, which could be less than the sum of the costs for  $x$  and  $y$  if sold individually. This bidding language is useful in markets with economies of scope, where suppliers have cost complementarities due to reduced production or transportation costs for a set of items.

Combinatorial auctions have been intensively discussed for the sale of spectrum licenses by the US Federal Communications Commission (FCC) [Milg00]. The FCC divides licenses into different regions. Currently, the simultaneous multiple round auction (SMR) is used to allocate these licenses, i.e., multiple auctions are conducted in parallel. Bidders, usually large telecom companies, have strong preferences for licenses that are adjacent to each other. This can have advantages in advertising a service to the end customer, but also in the infrastructure that needs to be set up. In simultaneous auctions, bidders risk that they only win one item from a set of items that they are interested in, but that they end up paying too much for this item. This is also called the exposure problem. These types of preferences can easily be considered in combinatorial auctions. In the mid-90's these auctions have, however, been considered impractical [McMi94] for at least two reasons:

- the computational complexity of the winner determination problem
- the strategic complexity for bidders

The allocation problem in combinatorial procurement auctions is a weighted set packing problem, a well-known NP-complete problem [VrSV03]. We will discuss this central problem in more detail in the next subsection. Strategic complexity for bidders has turned out to be another difficult problem. Bidders need to know their valuations for  $2^m - 1$  possible bundles of  $m$  items in the auction. Even if they know all valuations, they need to determine an optimal bidding strategy. Researchers have proposed different auction formats which exhibit various degrees of strategic complexity for bidders (see Section 3).

| Line | Bids                     | Gebote |      |      |      |
|------|--------------------------|--------|------|------|------|
|      |                          | B1     | B2   | B3   | B4   |
| 1    | 1000t Sugar in Munich    | 1      | 0    | 1    | 1    |
| 2    | 800t Sugar in Bonn       | 0      | 1    | 1    | 1    |
| 3    | 800t Sugar in Berlin     | 1      | 1    | 1    | 0    |
| 4    | Bid price (in thousands) | €150   | €125 | €300 | €125 |

Table 1: Example with bundle bids

## 2.1 The winner determination problem

First, we will concentrate on the winner determination problem in combinatorial procurement auctions. It is an excellent example of the types of optimization problems that one encounters in various multidimensional auctions. The following example with 4 bids and 3 items illustrates a simple setting. The buying organization needs different quantities of sugar in different production sites. In this case, the buyer aggregates demand for multiple production sites, as suppliers might be able to provide better prices due to reduced production and transportation costs. Suppliers bid on subsets of the demand and each subset has a bundle price (see Table 1).

The optimization problem can be formulated as a binary program. There are  $j \in L$  bids, and the set of items  $M$  indexed with  $k = 1, \dots, m$ . Each supplier  $i \in N$  submits a set  $L^i$  of bids. Each bid  $b_{ij}$  has a price  $p_{ij}$  and the set of items in a bid is described by a binary vector  $a_{ij}^k$ . If bid  $b_{ij}$  satisfies the entire demand of item  $k$ , then  $a_{ij}^k = 1$ , otherwise 0. The winner determination problem (WDP) can be formulated as follows:

$$\begin{aligned}
\min \quad & \sum_{i \in N} \sum_{j \in L^i} p_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i \in N} \sum_{j \in L^i} a_{ij}^k x_{ij} \geq 1 \quad \forall k \in M & (a) \\
& x_{ij} \in \{0, 1\} \quad \forall i \in N, \forall j \in L^i & (b)
\end{aligned}$$

The decision variable  $x_{ij}$  is 1, if bid  $b_{ij}$  is a winner in the auction, otherwise 0. Constraint (a) ensures that the total supply of items satisfies the demand. In other words, lines 1 to 3 in Table 1 are transformed into side constraints of the binary program. Line 4 will be transformed to the objective function. Note that the cost minimal solution can provide more items than the actual demand specified for the different sites.

This allocation problem is NP-complete, i.e., while the allocation problem in simple auctions is trivial, we do not know polynomial time algorithms for the winner determination problem in combinatorial auctions. Rothkopf and Pekec [RoPe98] analyze different approaches to limit the bidding language, in order to solve the allocation problem in polynomial time. Unfortunately, in every known case, the restriction required is so severe as to make the design impractical for any real-world auction. Independently, different algorithmic approaches for exact or heuristic solutions have been analyzed [VrVo03]. Existing methods from combinatorial optimization have been used, but also new algorithms have been developed [Sand99]. The problem sizes in many real-world applications have shown to be tractable. Most instances of the winner determination problem with several dozens of items and several hundred bids can be solved in a few seconds. For example, the combinatorial auction for Sears Logistics comprised 850 items [?]. Some papers have suggested meta heuristics for very large auctions with many bidders and many items. Optimality of an allocation, however, is important for the allocative efficiency of an auction.

In addition to the basic formulation of the winner determination problem, procurement applications often require additional side constraints such as:

- Purchasing managers want to specify a lower bound on the number of winners, in order not to become dependent on a single supplier. They also specify an upper bound to limit transaction costs due to too many suppliers.
- They also specify bounds on the quantity or volume purchased from a particular supplier or a group of suppliers (e.g., quotas for small- and medium-sized enterprises)

Side constraints of this sort can significantly impact the solution time for the winner determination problem [DaKa00].

## 2.2 Other types of multidimensional auctions

Apart from bundle bids, other types of complex bids have shown to be useful in procurement. Volume discount bids allow for bids specifying supply curves, i.e., unit prices for different quantities of an item sold [DaKa00]. With this type of bids supplier can express economies of scale, when bidding on very large quantities (e.g., €500/unit until 1000 units and €450/unit for more than 1000 units). Typically, buyers need to consider various business constraints when selecting such bids. For example, there might be limits on the spend per bidder or group of bidders, and upper and lower bounds on the number of winners. These side constraints turn the winner determination problem into a hard computational problem.

*Multi-attribute auctions* allow bids on price and qualitative attributes such as delivery time or warranty. In contrast to request for quotes or tenders as they are regularly used in procurement, the purchasing manager specifies a scoring function that is used to evaluate bids [Bich01]. This enables competitive bidding with heterogeneous, but substitutable offers. In addition to game theoretical models [Che93, Bran97], several implementations have been proposed and tested experimentally in the past few years [BiKS99, Bich00, BiK100, Stre03]. Multi-attribute auctions differ in the types of scoring rules or functions used, and in the type of feedback that is provided to bidders. These implementation specifics can have a significant impact on the auction results and are beyond what has been analyzed in game theoretical models. Depending on the type of bids submitted, and on the type of (linear or non-linear) scoring function, the auctioneer faces different optimization problems. For example, Bichler and Kalagnanam [BiKa05] describe allocation problems for *configurable offers* describing the price of an item as a function of attribute values. These types of bids allow for a compact representation of pricing policies (e.g., CPU *A* has a markup of €100 for a laptop, while CPU *B* has a markup of €150; the purchase of operating system *X* and office package *Y* implies a discount of €60; operating system *X* and office package *Z* are incompatible, etc.). These rules can be described in offers and automatically considered by the buyer in the bid evaluation or winner determination resp. The scoring function and the bid type have implications on the computational complexity of the allocation problem, but also on the strategic complexity for bidders. As of now, there is little research on these questions.

## 2.3 Desirable economic properties

Efficiency, revenue properties, and optimal bidding strategies of different auction formats are at the core of traditional auction theory [Wolf96]. So far, there has been a limited amount of game theoretical work on combinatorial and other types of multidimensional auctions. On the one hand, combinatorial auctions are a relatively young field. On the other hand, they are much harder to analyze game theoretically than single-item auctions.

Before we will discuss specific combinatorial auctions, we will introduce a number of desirable economic properties. Auction design describes the rules of an auction. These rules incentivize bidders to reveal their private valuations or costs resp. so that the auctioneer is able to determine the efficient allocation based on the bidders' true valuations.

Two types of goals are regularly discussed in the literature [Jack00]:

- *Allocative Efficiency* is achieved, if the auction leads to an allocation that maximizes the sum of all payoffs of the bidders and the auctioneer.
- *Revenue maximization* is achieved, if the auctioneer maximizes his revenue (or minimizes his cost in a procurement auction). This is often called "optimal" auction design.

*Incentive compatibility* and *strategy proofness* are properties that should incent bidders to reveal their private valuations. An auction is incentive compatible, if truthful revelation is a Bayes Nash equilibrium, i.e. truth revelation is optimal, if also all other bidders reveal their true valuations. An auction is strategy proof, if truth revelation is even a dominant strategy for bidders, i.e. it is the bidder's best strategy independent of other bidders' strategies. In these cases, the strategic complexity of an auction is reduced to a minimum and speculation is not necessary.

Apart from properties such as allocative efficiency or revenue maximization, *individual rationality* and *budget balance* are additional desirable properties. An auction mechanism is individually rational, if all participants have a positive expected utility. Budget balance requires that the auctioneer must not make a loss. There are proofs showing that it is impossible to have bilateral trading mechanisms that are efficient, budget-balanced, incentive compatible, and individual rational subject to certain assumptions [MySa83].

Game theoretical and experimental analyses of combinatorial and general multidimensional auctions are in their infancy. Due to the difficulties of analytical models in this field, many researchers have started with lab experiments on new auction designs and then derive useful theory from these observations, rather than start with theory development and then test it in the lab. These types of experiments are sometimes called "wind-tunnel experiments".

## 3 Combinatorial auctions

Apart from the type of bids allowed, a format describes the rules of the message exchange protocol. This protocol and the pricing rules determine the strategic complexity of auctions.

### 3.1 Sealed-bid auctions

One can distinguish between sealed-bid and open auctions. In sealed-bid auctions, bidders submit their bids to the auctioneer without additional information feedback until the auction closes. In open auctions, bidders can see information about what other bidders have bid. The first and the second-price sealed-bid auction (a.k.a. Vickrey auction) are well-known sealed-bid formats.

#### 3.1.1 The first-price sealed-bid auction

Some applications of combinatorial auctions are based on sealed-bid formats [ElKe02, EpHC02, RaSB82]. All bids need to be submitted until a particular end date, when the cost-minimizing combination of bids is selected. A number of properties of single-item sealed-bid auctions can also be observed in combinatorial auctions. First-price sealed-bid auctions are robust against collusion [Robi85]. However, the strategic complexity is quite high as compared to the Vickrey auction or generalizations of the English auction. For example, the bidder with the lowest cost for an item could speculate that others have higher cost and bid above his true cost. While he would increase revenue, it might also happen that he does not win. The strategic complexity arises in determining an optimal bid, given stochastic information about the cost distribution of competitors. In comparison, English auctions have a

simple dominant strategy of bidding down to the true cost, and then win or drop out of the auction.

### 3.1.2 The Vickrey-Clarke-Groves mechanism

Vickrey-Clarke-Groves (VCG) mechanisms describe a class of strategy-proof economic mechanisms [Vick61, Grov73], where sealed bids are submitted to the auctioneer. The principle can be applied to combinatorial auctions and is also called Generalized Vickrey auction (GVA). GVAs have a number of favourable properties, but also a few problems that should be discussed in the following.

Similar to single-item Vickrey auctions, bidders submit their private costs to the auctioneer. In a GVA, this means, a bidder needs to submit bids on all possible bundles. Each winning bidder receives a Vickrey payment, which is the amount that he has contributed to lowering the total cost of the buyer. Let's assume, two items  $x$  and  $y$  should be purchased. Supplier 1 bids €20 for  $\{x\}$  (i.e.,  $B_1(x) = €20$ ),  $B_1(y) = €11$  and  $B_1(x, y) = €30$ . Supplier 2 bids  $B_2(x) = €14$ ,  $B_2(y) = €14$  and  $B_2(x, y) = €26$ . The total cost will be minimized at €25, while purchasing  $\{x\}$  from supplier 2 and  $\{y\}$  from supplier 1. Supplier 1 demands €11 for  $\{y\}$ , but he receives a Vickrey payment of €26 - €25 = €1, since without his participation the total cost would be €26. In other words, the net payment of the buyer to supplier 1 is €12. Supplier 2 bids €14 for  $\{x\}$ , but receives a Vickrey payment of €30 - €25 = €5, because without his participation, the total cost of this auction would be €30. The advantage of VCG mechanisms is that they provide simple, dominant strategies for bidders. Unfortunately, they also exhibit a number of problems:

- The GVA requires bidders to submit bids on all  $2^m - 1$  possible bundles. Even if these bids will not enter the allocation, they can impact the Vickrey payments of bidders. Clearly, even with fairly small  $m$ , this can easily become impossible.
- The auctioneer needs to solve the allocation problem, which is NP-hard, but also determine Vickrey payments for each winner, which is again an NP-hard problem. All these problems need to be solved based on all possible bids of all bidders.
- In general, Vickrey auctions require a trusted auctioneer. An auctioneer could introduce synthetic bids to minimize Vickrey payments. Also, in a repeated setting, if the auctioneer is the buyer, he could use true information about the suppliers' costs in future auctions. Therefore, bidders will be reluctant in giving away their true cost. Cryptographic solutions to this problem have only been discussed for single-item Vickrey auctions [Bran03].

In addition, all arguments are based on the assumption of having independent private costs. In case of affiliated valuations, iterative auctions, such as the English auction, have been advocated by theorists [MiWe82].

## 3.2 Open-cry auctions

An open-cry auction enables bidders to learn about other bidders' cost [McMc87]. These mechanisms are typically run in multiple rounds, which is why we will refer to them as iterative auctions. The US FCC has almost exclusively considered iterative auctions for the design of spectrum licenses. They also seem to be attractive for industrial procurement applications [HoRR03].

### 3.2.1 General problems

Cramton [Cram98] summarizes a number of general arguments for iterative auction designs, which hold also for the case of combinatorial auctions. In combinatorial auctions, bidders have the possibility, to submit bids on bundles in later rounds, which they did not consider in the first round as they learn about the competition. In the past few years, a number of iterative combinatorial auction formats have been developed [BrP196, Park99, DeKL02, WuWe00, AuMi02]. Researchers try to develop mechanisms with minimal complexity for bidders and the auctioneer, without jeopardizing economic properties, such as allocative efficiency, strategy proofness, budget balance, or individual rationality. Some general problems of iterative combinatorial auctions should be discussed below.

**3.2.1.1 The threshold problem** describes the difficulty of small bidders to outbid a big bidder, who is interested to sell many items. Let's assume, a buyer wants to purchase three items  $x$ ,  $y$ , and  $z$ . Bidders 1, 2, and 3 would each be willing to sell one of the items for €3. Bidder 4 wants €10 for all three items (private valuation). In round 1, bidders 1, 2, and 3 have bid €4, while bidder 4 has already bid €10 for the bundle. None of the bidders 1, 2, and 3 can win, by lowering his bid to his private cost, and bidders need to coordinate. This difficulty is one of the reasons, why iterative auctions might end up with inefficient allocations.

**3.2.1.2 The exposure problem** is typically discussed for simultaneous auctions, such as SMR. When two items should be purchased, a bidder wants both and not only one of the items, he risks winning only a single item in these parallel events. Bidders often shade their bids, which is one of the reasons for inefficiencies in these simultaneous auctions. Bundle bids mitigate this problem.

A similar problem can, however, also occur in combinatorial auctions. Let's assume, two items  $x$  and  $y$  should be bought. If a bidder bids on  $x$  and  $y$  and wants only one of the items, he might end up with the bundle  $xy$  based on the winner determination formulation presented above. In particular, in iterative auctions, where bids are valid throughout the process, bidders want to make sure that they do not win multiple bids, although they only want one of them. An extension of this so called OR bidding language is the XOR bidding language, where only one of the bids of a bidder can become a winning bid. In this case, the auctioneer adds side constraint (c) to the WDP:

$$\sum_{j \in L^i} x_{ij} \leq 1 \quad \forall i \in N \quad (c)$$

As a consequence, bidders need to specify a larger number of bids for all combinations of items they are interested in. Logic bidding languages of this sort should allow bidders to describe their preferences easily. OR and XOR bidding languages are two examples. Also combinations and extensions have been discussed [Nisa00, FuLS99].

**3.2.1.3 Tie breaking:** In traditional auctions auctioneers typically choose the first of two equal bids. In combinatorial auctions, an allocation comprises multiple bids and an auction round might contain multiple allocations with the same total cost. The auctioneer needs to decide, whether he wants to choose the allocation that was possible first, or the allocation with the lowest average time stamp, or other rules and it obviously takes more effort to break ties [HoRR03].

**3.2.1.4 Ask prices:** In iterative auction formats, ask prices help bidders to determine, how much less to bid, in order to become a winner in the next round. In single-item auctions, a bidder needs to bid less than the current standing bidder. In combinatorial auctions, this is not obvious. If the current allocation of three items A, B, and C is that the bundle of item A and B go to supplier 1 and item C goes to supplier 2, the losing bidder 3, who is interested in the bundle of item B and C does not know, how much less to bid in the next round. This depends on the set of losing bids in the auction.

Bikhchandani and Ostroy [BiOs01] show in their fundamental contribution that in the presence of bundle bids, the only types of ask prices that are always possible are non-linear and personalized. In other words, there might be a price for every bundle and this bundle price might be different from bidder to bidder. The authors draw on linear programming duality theory in their work. This insight is fundamental. The second theorem of welfare economics states that any efficient allocation can be sustained by a Walrasian equilibrium, i.e., item-level or linear prices [MaWG95]. In combinatorial auctions, some of the assumptions in welfare economics are violated and one cannot always find linear ask prices. Obviously, the sheer volume of non-linear and personalized ask prices is a disadvantage. Nevertheless, some researchers have proposed practical auction designs based on this type of prices. Others have decided to approximate linear ask prices. The type of ask prices is the main difference among practical combinatorial auction designs.

### 3.2.2 Selected combinatorial auction designs

A number of designs for iterative combinatorial auctions have been proposed recently. The discussion on the design of the US FCC spectrum auctions was certainly one of the drivers for this increased interest. The Adaptive User Selection Mechanism (AUSM) was one of the first proposals [BaLP89]. Here, the allocation problem is delegated to bidders and a public white board should help them to coordinate their bundle bids and suggest new composite bids that have a lower total cost for the buyer. Most newer auction designs have the auctioneer calculate the optimal allocation [WeGS99, DeKL02, PoRR03, Park99, WuWe00, AuMi02, VrSV03]. In the following, we will provide a brief overview of the main proponents:

The *Resource Allocation Design (RAD)* approximates linear ask prices and combines them with a number of activity rules from SMR. RAD requires monotonicity in the number of items a bidder demands. This means, in each round a bidder can only bid on as many items in his bids as he bid on in the previous rounds. The ask price for a bundle is simply the sum of the item prices in this bundle. Bidders need to bid below the ask prices minus the bid decrement (in reverse auctions). The LP-based heuristic to calculate ask prices is specific to RAD [DeKL02].

An alternative and simple approach has been proposed by Porter et al. [PoRR03]. The *combinatorial clock auction (CC)* proceeds like a Japanese auction on multiple items. For each item, there is an item clock showing the current ask price per item. In each round the bidder determines which bundles he wants to bid on at the current prices. Ask prices for bundles are again linear. The clock starts at a very high price and is decreased round by round as long as there is competition on this item, i.e., as long as there is more than one bidder interested in selling the item. After a certain number of auction rounds, there should be linear ask prices determining the allocation. There might be cases, in which the ask prices are too low to have any bidder any more. In these cases, the auctioneer solves the WDP based on all bids submitted in the auction. In experimental analyses the CC auction led to higher efficiency compared to SMR in cases with cost complementarities. As in RAD, the linear ask prices used in the CC auction can lead to inefficiencies.

A number of scientists have tried to address the inefficiencies due to linear prices by using non-linear ask prices [Park99, WuWe00, AuMi02, VrSV03]. These auctions are sometimes referred

to as primal-dual auctions, as the way how the efficient solution is found can be modelled similar to primal-dual or subgradient algorithms in linear programming [NeWo88]. Here, the dual variables are interpreted as ask prices in an auction. iBundle is one of these auction designs [PaUn00]. Prices for each losing bundle and bidder are decreased by a minimum bid increment from round to round. Assuming straightforward bidding, this means that every bidder submits bids on those bundles that maximize his payoff, this auction format leads to efficient allocations. Ausubel and Milgrom have suggested the use of proxy agents, in order to deal with the many auction rounds that this format causes [AuCM03].

## 4 Applications in industrial procurement

Companies such as CombineNet (<http://www.combinenet.com>), NetExchange (<http://www.netex.com>) and Trade Extensions (<http://www.tradeextensions.com>) provide software for combinatorial auctions. Although there are a number of press announcements about the use of combinatorial auctions for transportation and industrial procurement, there are only a few published cases. We have analyzed the literature and conducted a number of phone interviews with representatives of these three companies. Based on this initial survey, combinatorial auctions have been used for the procurement of very different types of items (office supply, chemicals, transportation services, package material, etc.). In most cases there are larger quantities of items with a high degree of cost complementarities for suppliers. Vendors reported auctions with 10 items, but also cases with many thousands items. Similarly, the number of bidders varied. There were cases with several hundred bidders; 10 to 20 bidders were seen as average numbers for industrial procurement applications, however. Even though the number of empirical observations is limited, the results of these interviews illustrate the versatility of combinatorial auctions as a means for online negotiations in industrial procurement.

Procurement applications have been conducted both, as sealed-bid and as iterative auctions. Iterative auctions were typically run without ask prices, providing bidders only information about the winning allocation. We did not learn about any use of VCG mechanisms in this domain. What has been described as essential to procurement applications by all respondents was the consideration of additional side constraints such as upper and lower bounds on the number of winners or spend on a supplier. Also, some vendors used extensions of pure bundle bids including qualitative attributes and quantity of an item.

*Cost savings* were seen as the main motivation for the use of combinatorial auctions in procurement organizations. According to the vendors, combinatorial auctions delivered on their promise and they quoted cost savings of up to 13 % on average. This can mostly be attributed to higher allocative efficiency of these auctions. In particular in cases with significant cost complementarities for suppliers, combinatorial auctions led to high allocative efficiency compared to other types of auctions [PoRR03]. A few of reasons were mentioned in addition to reductions in total cost:

**Decreased transaction costs for complex procurement negotiations:** Combinatorial auctions allow for effective negotiations on multiple items. The alternative to combinatorial bidding are either sequential or simultaneous auctions or bilateral negotiations on the phone, which is time-consuming and expensive (see also [HoRR03]).

**Transparency and fairness:** Open and iterative auctions increase the market transparency, which was seen as a positive feature by suppliers. Also, all suppliers are treated equally, which led to a high perceived fairness by the suppliers.

## 5 Summary

The US FCC spectrum auctions have spawned intensive discussion on the design of combinatorial auctions. Overall, however, industrial procurement might be one of the most interesting application domains. Cost savings for the purchasing manager, decreased transaction costs, transparency, and fairness are among the main advantages. However, the application of combinatorial auctions leads to a number of fundamental design problems such as the computational complexity of the winner determination problem or the strategic complexity for bidders. In addition, procurement applications have specific requirements such as the consideration of additional side constraints and the need for other, multidimensional bid types. A number of promising auction designs has been proposed, but to date, a solid theoretical and empirical evaluation of these designs is missing. In the long run, this research area might lead to robust and efficient auction designs and standard software solutions for multi-item negotiations in procurement.

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